EXAMPLE TURING MACHINE

Turing machine accepting $\{0^n 1^n : n \geq 1\}$

$X, Y \notin \{0, 1\}$

000111

$\rightarrow$

X00111

X00111

X00Y11

$\leftarrow$

X00Y11

X00Y11

XX0Y11

XX0YY1

⋮

XXXYYY␣

$\Gamma = \{0, 1, \vdash, \sqcup, X, Y\}$

State	$\vdash$	0	1	X	Y	$\sqcup$
s	$(q_1, \vdash, R)$					
q_1		(q_2, X, R)			(q_4, Y, R)	
q_2		$(q_2, 0, R)$	(q_3, Y, L)	$(r, -, -)$	(q_2, Y, R)	$(r, -, -)$
q_3		$(q_3, 0, L)$		(q_1, X, R)	(q_3, Y, L)	
q_4			$(r, -, -)$		(q_4, Y, R)	$(t, -, -)$

q_1 : moving right looking for the first 0

q_2 : moving right looking for the leftmost 1

q_3 : moving left looking for rightmost X

q_4 : scanning right skipping all Y's until $\sqcup$ is reached

Moves of the TM on input 0011

$$(s, \vdash 0011, 0) \xrightarrow[M]{1} (q_1, \vdash 0011, 1) \xrightarrow[M]{1} (q_2, \vdash X011, 2)$$

$$\xrightarrow[M]{1} (q_2, \vdash X011, 3) \xrightarrow[M]{1} (q_3, \vdash X0Y1, 2)$$

$$\xrightarrow[M]{1} (q_3, \vdash X0Y1, 1) \xrightarrow[M]{1} (q_1, \vdash X0Y1, 2)$$

$$\xrightarrow[M]{1} (q_2, \vdash XXY1, 3) \xrightarrow[M]{1} (q_2, \vdash XXY1, 4)$$

$$\xrightarrow[M]{1} (q_3, \vdash XXYY, 3) \xrightarrow[M]{1} (q_3, \vdash XXYY, 2)$$

$$\xrightarrow[M]{1} (q_1, \vdash XXYY, 3) \xrightarrow[M]{1} (q_4, \vdash XXYY, 4)$$

$$\xrightarrow[M]{1} (q_4, \vdash XXYY\sqcup, 5) \xrightarrow[M]{1} (t, \vdash XXYY, 6)$$

Techniques for TM Construction

1. Storage in finite control.

Consider language $\{ww^R : w \in \{0,1\}^*\}$

$$\overset{\downarrow}{\underline{a_1}}\,\underline{a_2} \ldots a_n\, a_n\, a_{n-1} \ldots \underline{a_2}\,\underline{a_1}$$

$Q : \{s, t, r, q_1\} \cup (\{q_2, q_3, q_4\} \times \{0, 1\})$

eg: $[q_i, 0]$

	0	1	⊔
q_1	$([q_2,0], ⊔, R)$	$([q_2,1], ⊔, R)$	$(t, -, -)$
$[q_2, 0/1]$	$([q_2,0/1], 0, R)$	$([q_2,0/1], 1, R)$	$([q_3,0/1], ⊔, L)$
$[q_3, 0]$	$([q_4, 0], ⊔, L)$	$(r, -, -)$	$(r, -, -)$
$[q_3, 1]$	$(r, -, -)$	$([q_4, 1], ⊔, L)$	$(r, -, -)$
$[q_4, 0/1]$	$([q_4,0/1], 0, L)$	$([q_4,0/1], 1, L)$	$(q_1, ⊔, R)$

2. Multiple tracks

a_1	a_2	a_3	. . . .
$\sqcup$	$\sqcup$	$\sqcup$	. . .
$\sqcup$	$\sqcup$	$\sqcup$	. . .

Γ^3 : tape alphabet.

each track could be used for a different purpose (e.g. i/p track, work tracks, ...)

3. Checking off symbols

$\{0^n 1^n : n \geq 1\}$

$\check{0}011 \rightarrow \check{0}0\check{1}1 \rightarrow \check{0}\check{0}\check{1}1$

$\rightarrow \check{0}\check{0}\check{1}\check{1}\sqcup \rightarrow$ accept

useful for checking repeated strings or comparing lengths of substrings

Checked symbols are separate symbols in Γ.
Check marks could be visualised to be on a separate track.

$\vdash$	✓		✓	
	0	0	1	1

4. <u>Shifting over</u>

Shift all symbols to the right by 1 position

$$\vdash a_1 a_2 \cdots a_n$$

move to rightmost non-blank symbol

$$\vdash a_1 a_2 \cdots a_{n-1} a_n \sqcup$$

$$\vdash \sqcup a_1 a_2 \cdots a_{n-1} a_n$$

TM that increments input binary number

$\vdash 1011$ $\quad \vdash 1010$ $\quad \vdash 1000$ $\quad \vdash 1100$

$\vdash 11 \rightarrow \vdash 00$

5. Subroutines

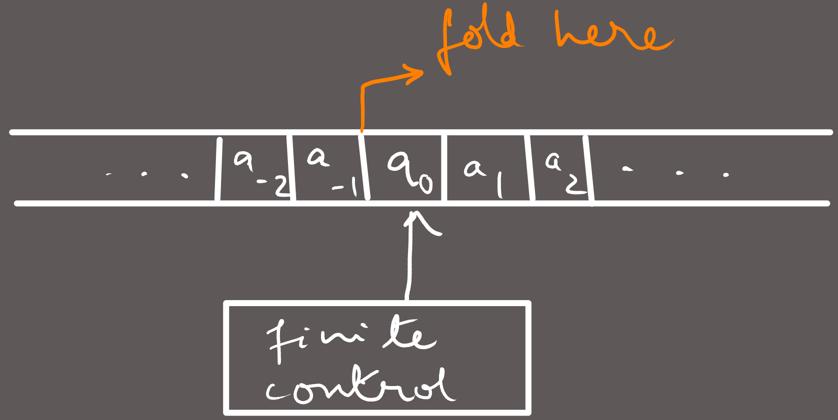
E.g: M_1: TM that shifts its i/p 1 position to the right.

M_1 can be used as a subroutine while designing a TM that increments its i/p or adds 2 given integers, ...

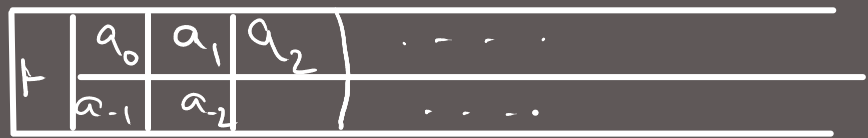
M: computes some function

VARIANTS OF TURING MACHINES

1. 2-way infinite tape

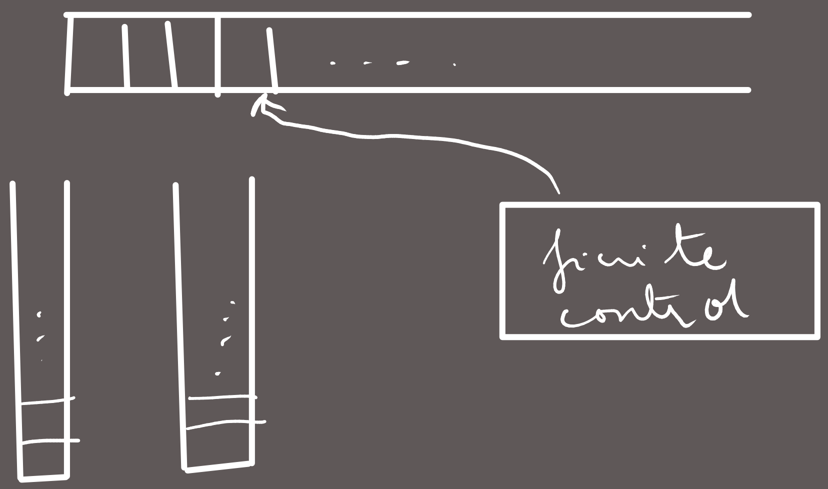


Fold the tap at some point



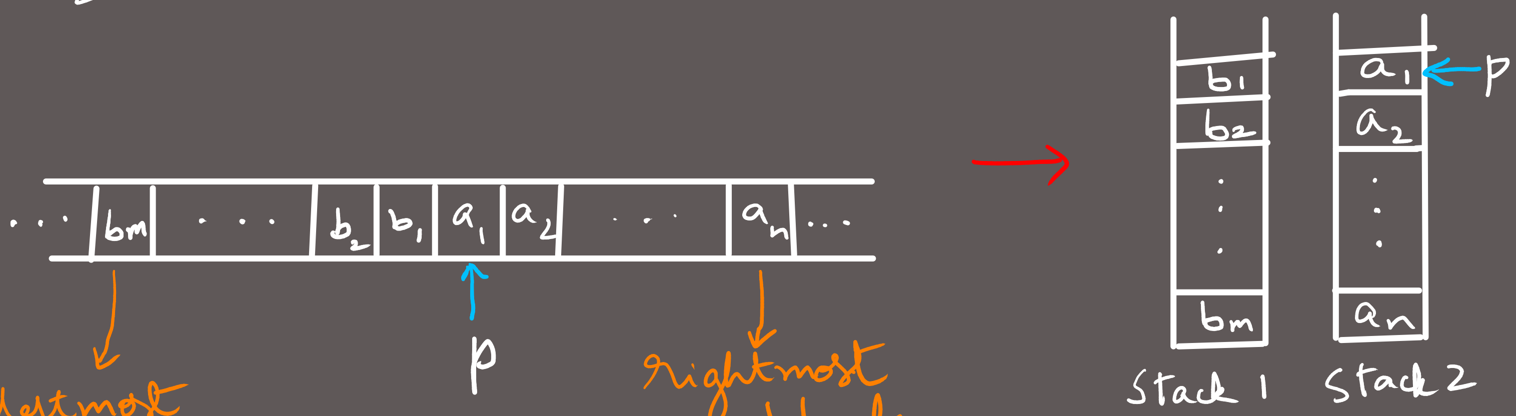
simulates the 2-way infinite tape.

2. Machine with 2 stacks & a 2-way read-only tapehead.



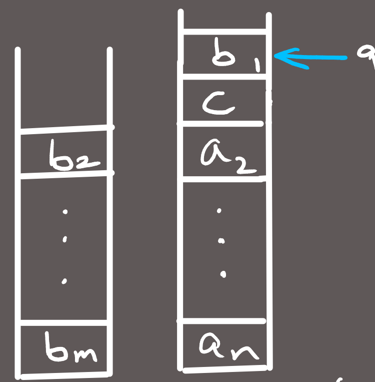
equivalent to Turing machines

2-stack m/c can simulate a 2-way infinite tape TM



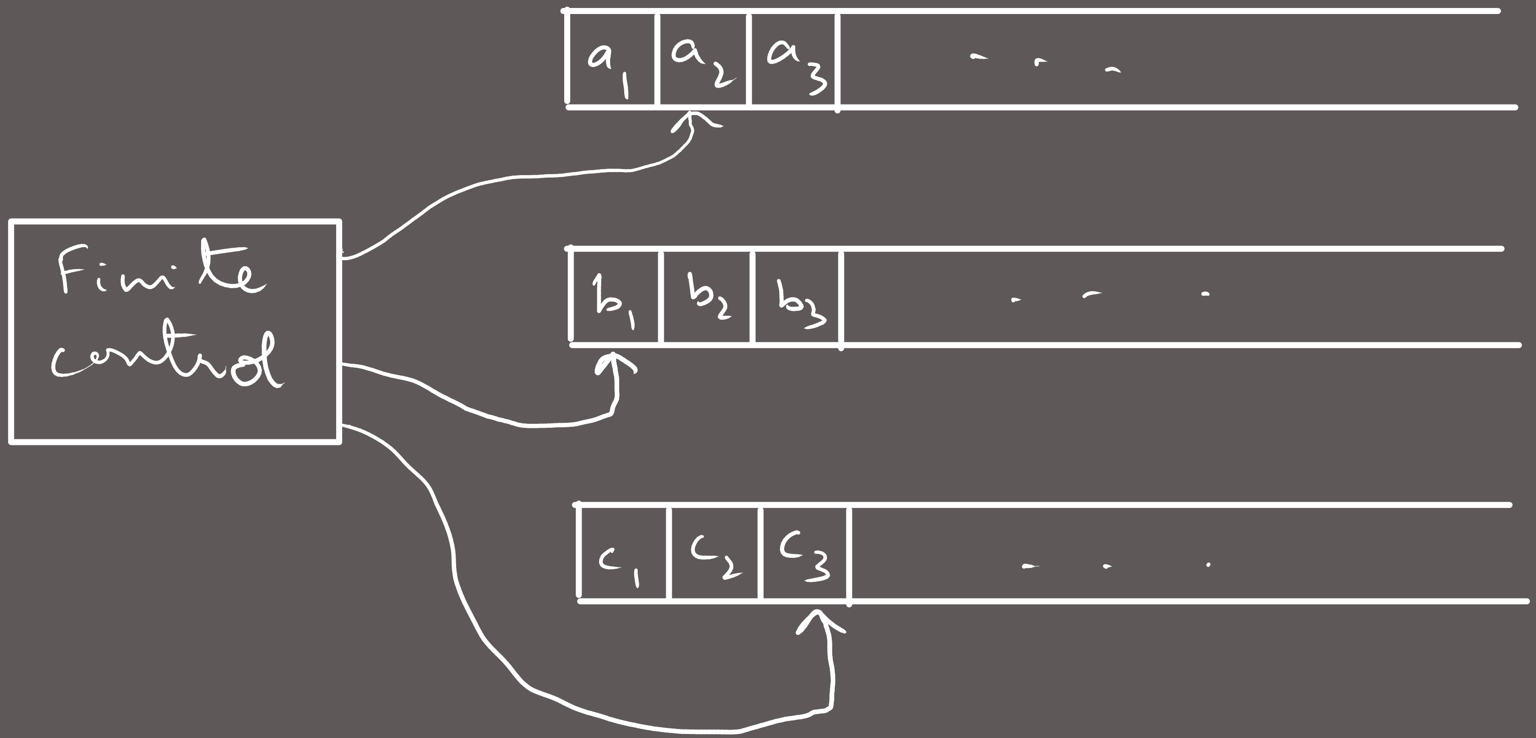
top of Stack 2 ≡ tape head position

Upon transition $\delta(p, a_1) = (q, c, L)$



$pop_2(\,),\ push_2(c),\ push_2(pop_1(\,))$

3. Multiple Tape Turing machines

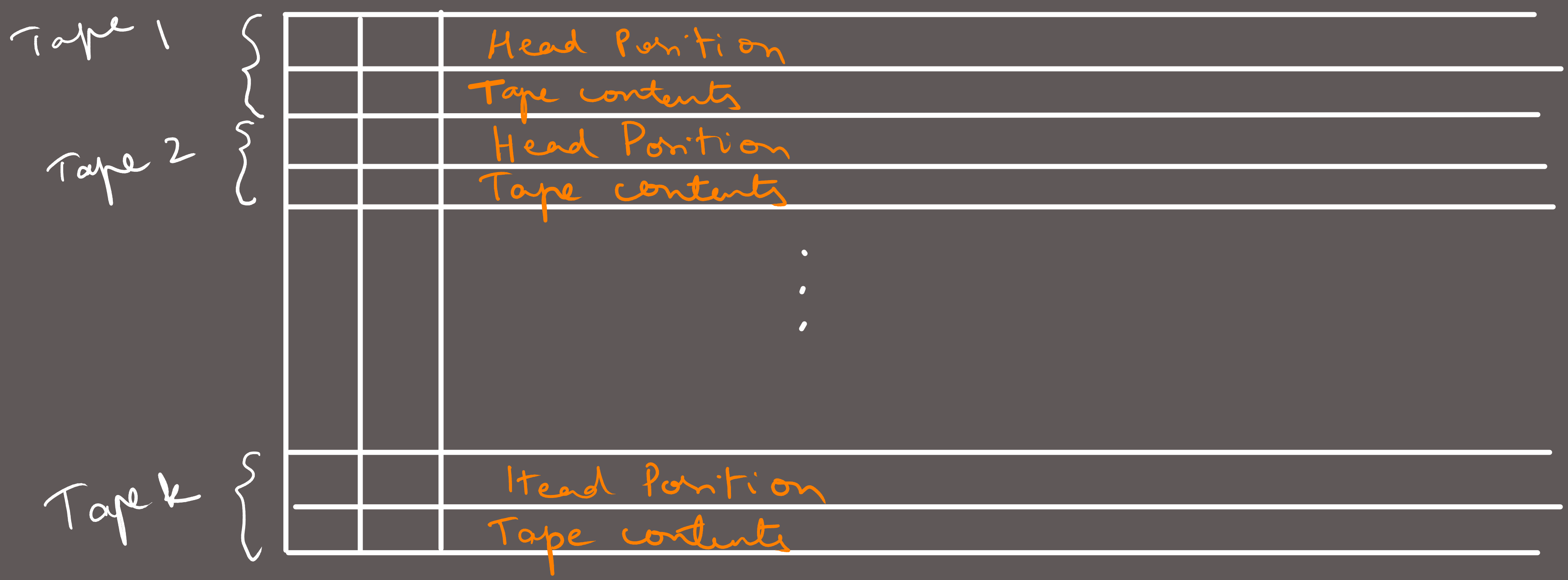


$k = 3$

k-tape TM : k tapes & k independent tape-heads

$$\delta : Q \times \Gamma^k \longrightarrow Q \times \Gamma^k \times \{L, R\}^k$$

Simulation of a k-tape TM by a single tape TM



4. k-Counter Automata

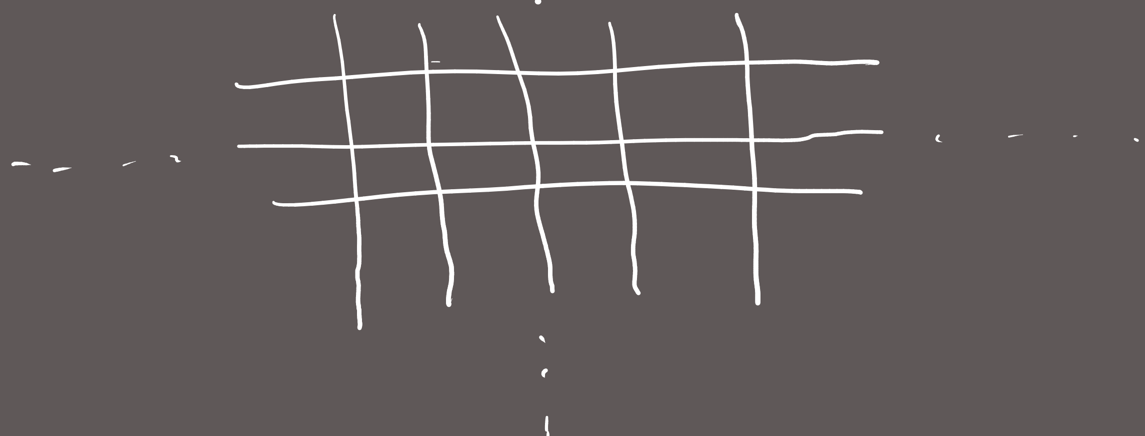
Read only i/p tape + k integer counters → contain a non-negative integers

increment / decrement / test for 0

One-stack m/c can be simulated using 2 counters

TM can be simulated using 4 counters.

5. Multi-dimensional tape TM



k-dimensional TM

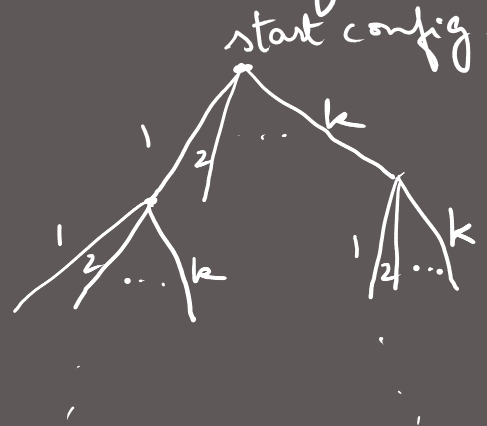
tape is infinite in all 2k directions.

equivalent to TMs.

6. Non-deterministic Turing machines.

Starting from a particular configuration, there are finitely many transitions possible.

k : max. no. of transitions possible.



Non-determinism does not make TMs more powerful.

2, 2, 3, k, k-4, ...

Simulating NDTM using DTM

- write down i/p of the NDTM on one track

repeat forever {
- write down finite sequence of integers from {1, 2, ..., k} on another track
- use a third track to simulate NDTM following transitions indicated by the sequence on second track
- if NDTM accepts, then accept & halt
}