Context-Free Languages:

SUMMARY

↳ Grammar $G = (N, \Sigma, P, S)$

Production rules → $P \subseteq N \times (N \cup \Sigma)^*$

- N: Set of non-terminals
- Σ: terminal symbols
- S: start symbol (non-terminal)

finite representation for infinite set of strings

↳ Derivations: $S \xrightarrow[G]{*} x$ (sentential form)

$\alpha \xrightarrow[G]{n} \beta$ iff $\alpha \xrightarrow[G]{n-1} \gamma$ and $\gamma \xrightarrow[G]{1} \beta$

▷ Ex: ① $L_1 = \{a^n b^n \mid n \geq 0\}$

$S \to aSb \mid \epsilon$

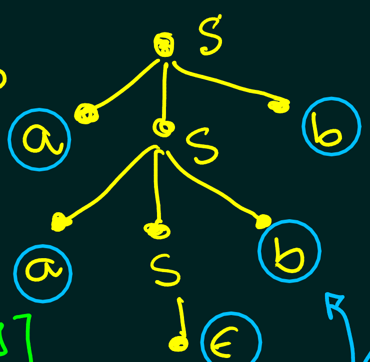
② L_2 = balanced parantheses

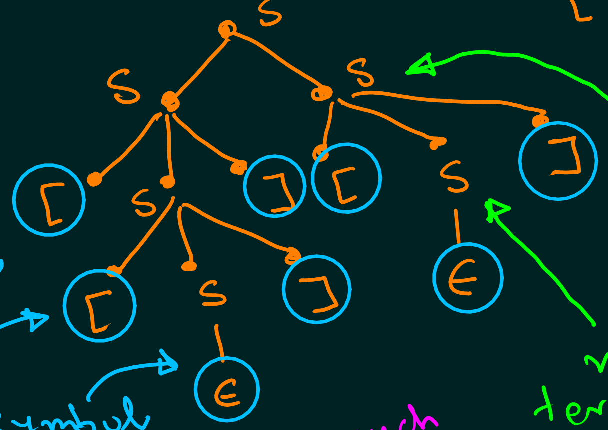
$S \to [S] \mid SS \mid \epsilon$

↳ Derivation/Parse Tree:

aabb in $a^n b^n$

[Ex #1]





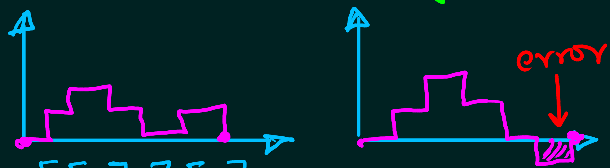
[[ ]] [ ] in balanced Parantheses [Ex #2]

▷ Proof of Correctness:

$S \xrightarrow[G]{*} x$ is balanced

iff ① $L(x) = R(x)$ and

② $L(y) \geq R(y)$, $y = \text{pref}(x)$



✓ [[ ]] [ ]   ✗ [[ ]]] [

↳ Leftmost Derivation → If 2-such AMBIGUOUS

$S \to SS \to [S]S \to [[S]]S$

$[[\epsilon]][\epsilon] \leftarrow [[\epsilon]][S] \leftarrow [[\epsilon]]S$

↳ Rightmost Derivation

$S \to SS \to S[S] \to S[\epsilon]$

$[[\epsilon]][\epsilon] \leftarrow [[S]][\epsilon] \leftarrow [S][\epsilon]$

▷ Normal Forms:

↳ Chomsky NF: $\begin{cases} A \to BC \\ A \to a \end{cases}$ (CNF)

equiv

↳ Greibach NF: $A \to a\alpha$, $(\alpha \in N^*)$ (GNF)

▣ Any Grammar → CNF → GNF

① $A \to \alpha B \gamma$ and $B \to \epsilon \Rightarrow A \to \alpha\gamma$

② $A \to B$ and $B \to \beta \Rightarrow A \to \beta$

CFL PDA

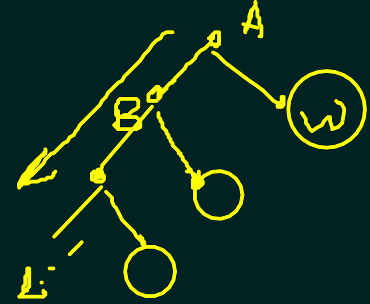
Regular Grammar :

$$\boxed{L = 0(10)^*}$$

$RL \subsetneq CFL$

↳ left linear: $A \to Bw$ where $w \in \Sigma^*$ $(w = w_1 w_2 \ldots w_k)$, $w_i \in \Sigma$

↳ right linear: $A \to wB$

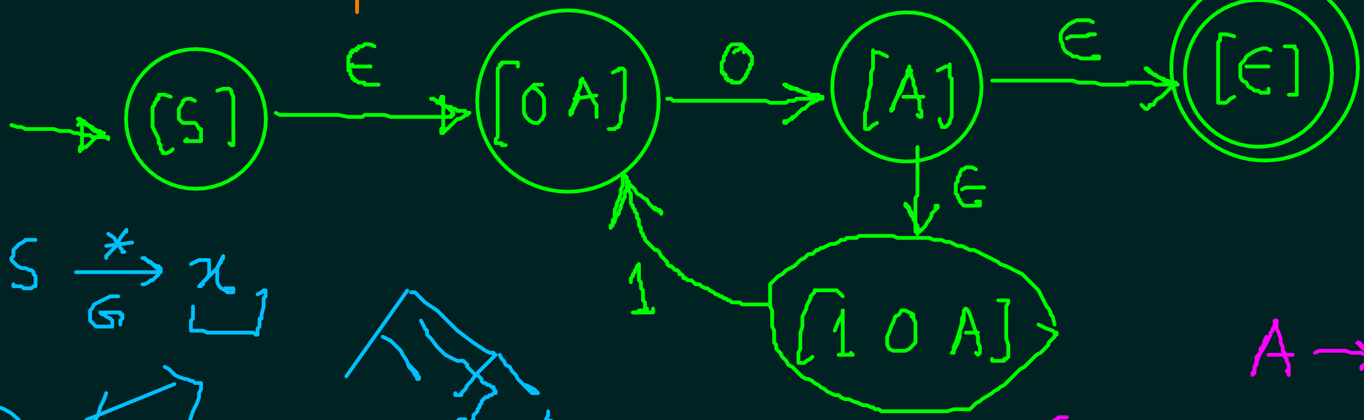


G_L : ✓ $S \to S10 \mid 0$ ✓

$\boxed{G_R}$: ↳ $S \to 0A$
$A \to 10A \mid \epsilon$

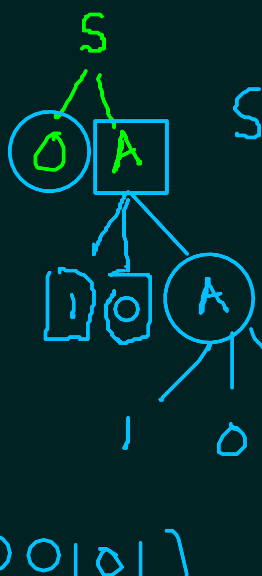
$Q = \{[S], [A], [0A], [10A], [\epsilon]\}$

NFA



① $[w_1 w_2 w_3 A]$
② $[w_2 w_3 A]$
③ $[w_3 A]$ ④ $[A]$

$A \to w_1 \overbrace{w_2 \ldots w_k}^{w'} B$



$S \xRightarrow[G]{*} x$

Rule of gen NFA

$[A] \xrightarrow{\epsilon} [w_1 w_2 \ldots w_k B]$

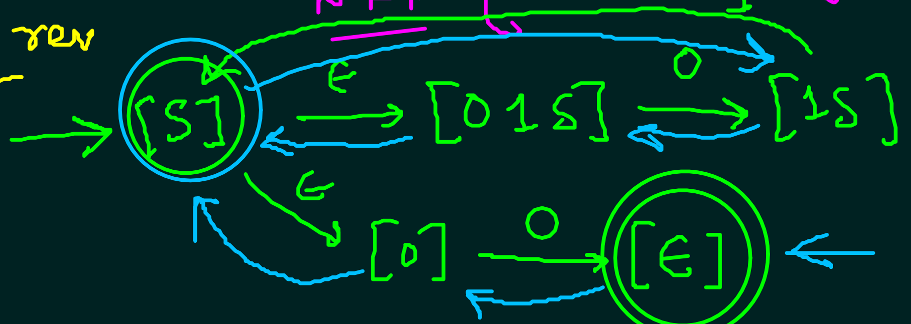
$[w_1 w' B] \xrightarrow{w_1} [w' B]$

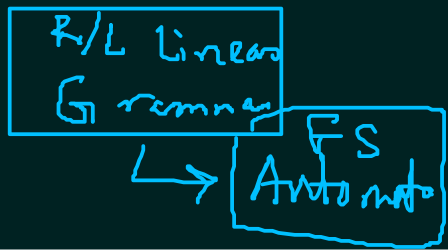
$[0\epsilon] \xrightarrow{0} [\epsilon]$

(00101)

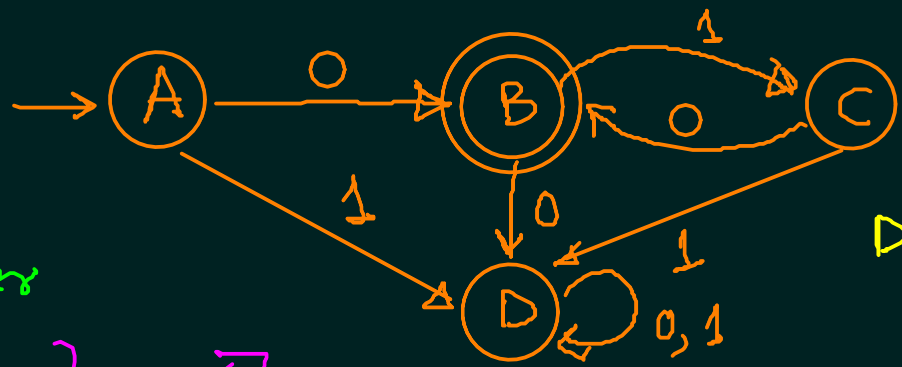
$\xrightarrow{NFA} L^{rev}$

$G_L^{rev} : \underbrace{S \to 01S \mid 0}_{R.L\ Grm}$





$L = 0(10)^*$



DFA $\longrightarrow$ R.L. Grammar

$A \to 0B \mid \cancel{1D} \mid 0$

$B \to 1C \mid \cancel{0D}$

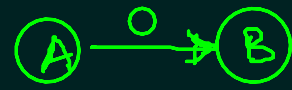
$C \to 0B \mid \cancel{1D} \mid 0$

$\boxed{D \to 0D \mid 1D}$

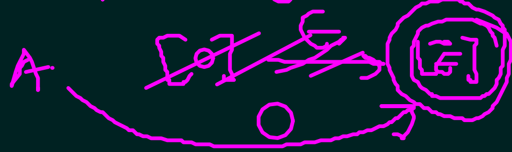
$$\boxed{\begin{array}{l} A \to 0B \mid 0 \\ B \to 1C \\ C \to 0B \mid 0 \end{array}}$$

DFA

$A \to 0B$



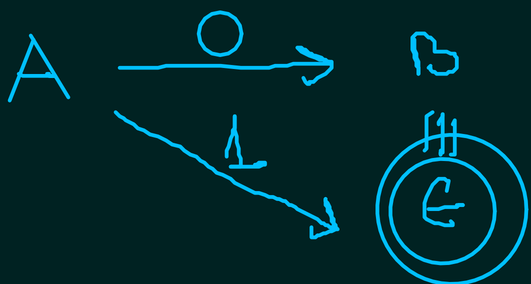
$A \to 0E$



Right linear Grammar

$\boxed{H/W}$: How can you produce left-linear Grammar?

1



$\equiv$

$A \longrightarrow 0B$

$A \longrightarrow 1\epsilon \equiv A \to 1$

2 $A \to w_1 w' B \equiv [A] \xrightarrow{w_1} [w'B]$

Pumping Lemma for CFL : "Not Context-free"

↳ Contr.

$G \rightsquigarrow \boxed{S \xrightarrow[G]{*} z}$

(N, Σ, P, S)

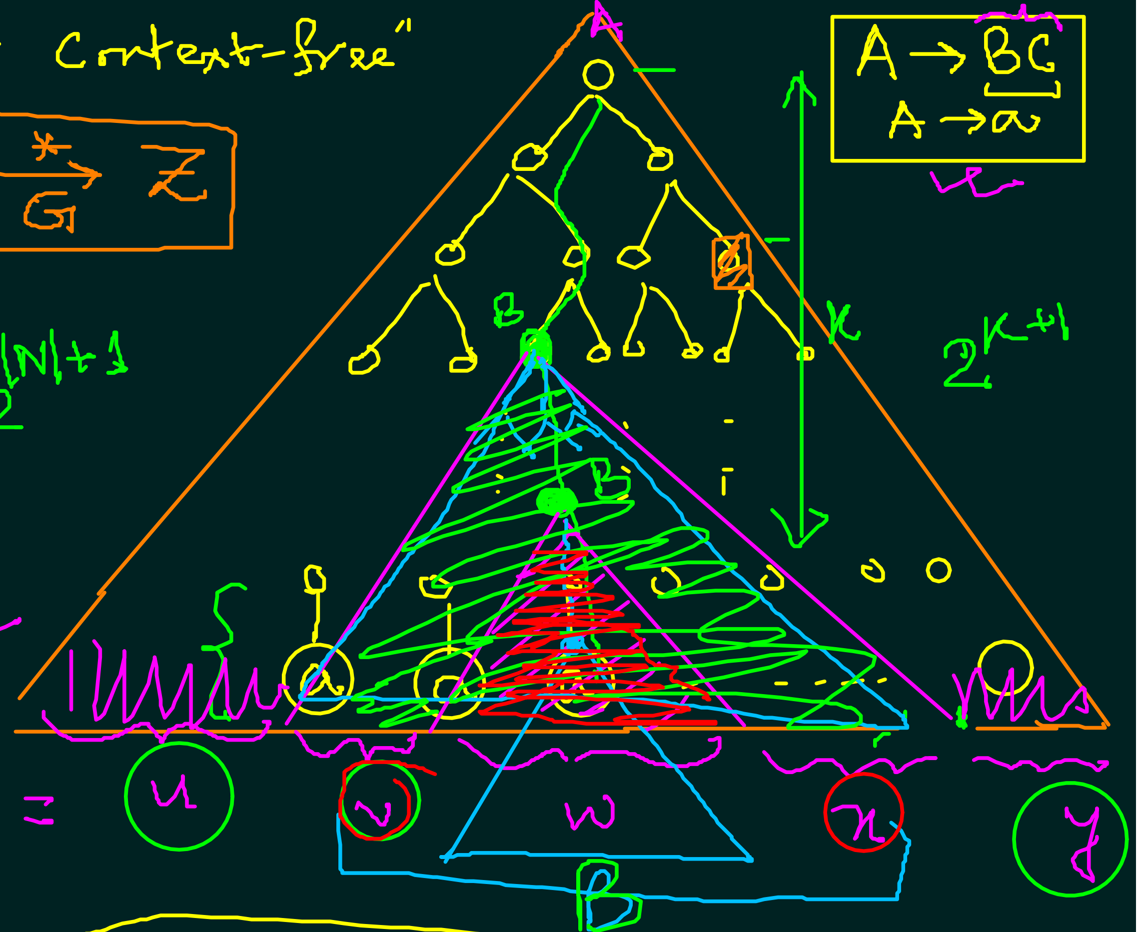
$|z| \geq 2^{|N|+1}$

$> |N|$

$> |N+1$

$\boxed{\begin{array}{l} A \to BC \\ A \to a \end{array}}$

2^{K+1}



$z = u\ v\ w\ x\ y$

$uvwxy \in L$

then $uv^iwx^iy \in L$

$(i \geq 0)$

for some i, $uv^iwx^iy \notin L$

Contr.

$A \xrightarrow[G]{*} uBy$

$B \to vBx$

$B \to vvBxx$

w

$uwy \in L$

↳ → $uvvwxxy \in L$

↳ → $uv^3wx^3y \in L$

Example (Application of PL for CFL):

$L = \{a^i b^i c^i \mid i \geq 1\}$

$z = a^n b^n c^n \longleftarrow n \qquad |z| \geq n$

$aa \ldots a\, bb \ldots b\, cc \ldots c$

(3) $z = uvwxy = a^n b^n c^n$

✓ (2) $|vwx| \leq n$

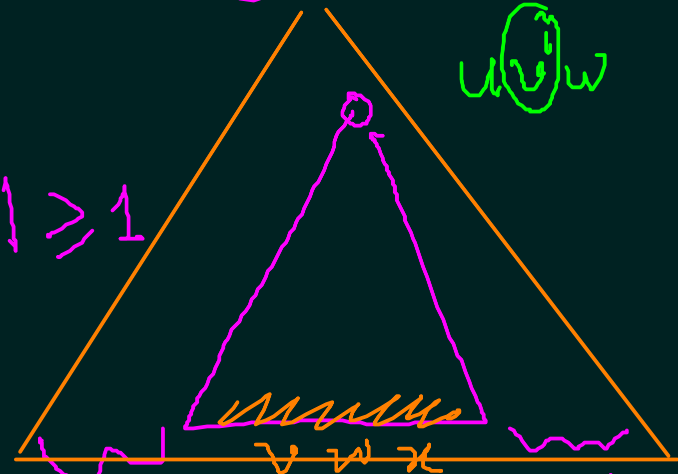
$\hookrightarrow vwx = a^n b^n c^n$ ($\leq n$)

$u (v)^i w (x)^i y \notin L$

$u \to \epsilon$ (4) ✓ $\qquad y \to \epsilon$

(1) $|vy| \geq 1$ ✓

$u (v) w$



$u \neq \epsilon$ — or — $y \neq \epsilon$

$vwx \longrightarrow$ full a (1) ✓ $\to a^l$

full b (2) ✓

full c (3) ✓

a, b (4) ✓

b, c (5) ✓

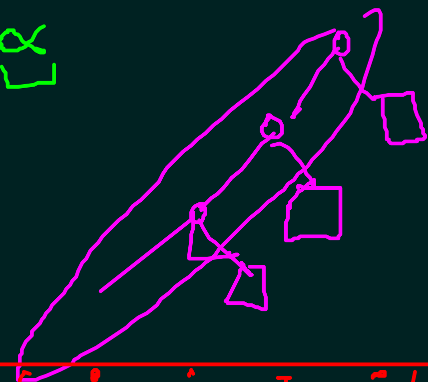
$a^m b^l$

~~$a^n b c$~~ ✗

$S \to y c \alpha$

$a^{n+l} b^n c^n \notin L$

$uv^2wx^2y = a^{n+m} b^{n+l} c^n \notin L$



$L' = \{a^i b^j c^i d^j \mid i, j \geq 1\}$ NOT CF

Closure Properties of CFL:

$G_1 = (N_1, \Sigma, P_1, S_1)$ ✓

$G_2 = (N_2, \Sigma, P_2, S_2)$ ✓

↳ union

$G = (N_1 \cup N_2 \cup \{S\}, \Sigma, P, S)$

$P = \underbrace{P_1} \cup \underbrace{P_2} \cup \{S \to S_1 | S_2\}$

↳ Concat. $P = P_1 \cup P_2 \cup \{S \to S_1 S_2\}$

↳ Kleene Star G_1^*

$\overline{A \cup B} = \bar{A} \cap \bar{B}$ ✓

$G = (N_1 \cup \{S\}, \Sigma, P, \{S\})$

$P = P_1 \cup \{S \to S_1 S | \epsilon\}$

↳ Intersection is NOT closed

$\{\underbrace{a^i b^i} c^j \mid i,j \geq 1\} \cap \{a^i b^j c^j \mid i,j \geq 1\} \leftarrow S \to A S_1$

CF $\quad$ CF

$S \to S_1 C$ $\quad G_1$? $\quad G_2$?

$= \underbrace{\{a^i b^i c^i \mid i \geq 1\}}_{\text{NOT CF}}$

→ Complement NOT closed