Regular Expression → NFA with ϵ-moves → NFA → DFA → Minimum State DFA (unique) → Regular Expression

- Regular Expression → NFA with ϵ-moves: Concat, union & Kleene Star
- NFA with ϵ-moves → NFA: ϵ-closure
- NFA → DFA: Subset Construct.
- DFA → Minimum State DFA: Equivalence Partition (Elim. Unreach)
- Minimum State DFA → Regular Expression: Recursive Formulation

[Regular Language]

SUMMARY

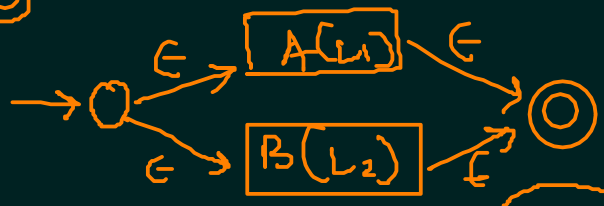
▷ Properties of Regular Languages: L_1 and L_2

Lang. is Regular → NFA ✓, DFA ✓, Reg Exp., Linear Gram.

↳ Concat $(L_1 L_2)$ is regular.

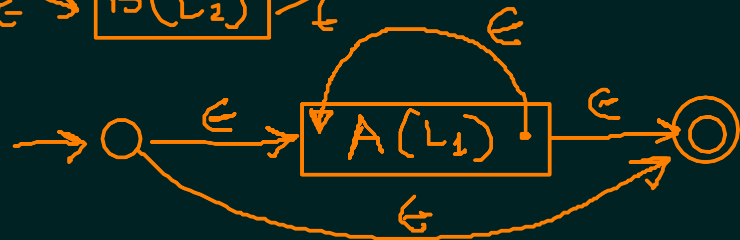
$\leftarrow L_1 L_2$

↳ Union $(L_1 \cup L_2)$ is regular



$L_1 \cup L_2$

↳ Kleene Star (L_1^*) is regular.



L_1^*

↳ Complement $(\bar{L}_1)$ is regular

$\bar{L}_1 = \Sigma^* - L_1$

$M(L_1) = (Q, \Sigma, \delta, q_0, F)$

$M'(\bar{L}_1) = (Q, \Sigma, \delta, q_0, Q-F)$ → regularity of $\bar{L}_1$

$0101 \in L_1$

$1010 \in rev(L_1)$

↳ Intersection $(L_1 \cap L_2)$ is regular → $L_1 \cap L_2 = \overline{\bar{L}_1 \cup \bar{L}_2}$

$M_1(L_1) = (Q_1, \Sigma, \delta_1, q_1, F_1)$

$M_2(L_2) = (Q_2, \Sigma, \delta_2, q_2, F_2)$

$M(L_1 \cap L_2) = (Q_1 \times Q_2, \Sigma, \delta, [q_1, q_2], F_1 \times F_2)$

$\delta([p, q], a) = [p', q']$

s.t. ${}_{M_1}\delta(p, a) = p'$ ${}_{M_2}\delta(q, a) = q'$

↳ $rev(L_1)$ is regular ??

Not all Languages are Regular: Prove Pumping Lemma → Contradiction

Ex: $L = \{a^n b^n \mid n \geq 0\}$ is ~~not~~ regular (assume) ✓

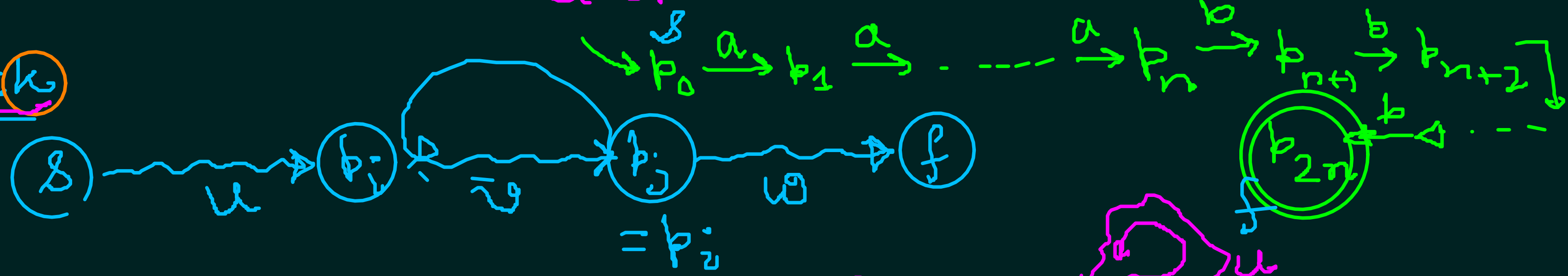
↳ 'K' state DFA

$n = \lceil K/2 \rceil \geq K/2$ $a^{\lceil K/2 \rceil}$ $b^{\lceil K/2 \rceil}$

accept.

$2n+1 \geq K+1 > K$

$a^n b^n = uvw$



$\underbrace{aa \ldots}_{v\ ①}\ \underbrace{abb}_{v\ ③}\ \underbrace{\ldots b}_{v\ ②}$

$1 \leq |v| \leq K$

$\boxed{uw \in L\ ??}$

$v = a^l$

$v = b^l$

$v = a^l b^l$

① $a^{n-l} b^n \notin L$ ✓

② $a^n b^{n-l} \notin L$ ✓

③ $a^{n-l} b^{n-l} \in L$ → Cannot be contr.

① $uw \in L$

② $uvvw \in L$

$uvvvw \in L$

$uv^i w \in L$, $i \geq 0$

$\boxed{uv^2 w \in L\ ?}$

$\underbrace{a}_{u}\ \underbrace{\boxed{a^{l}}}_{v}\ \underbrace{a n bb \ldots b}_{w}$

$a^i = \underbrace{aa \ldots a}_{i}$

$\underbrace{aa \ldots a}_{u}\ \underbrace{aaabb}_{v}\ldots\underbrace{aa \ldots b}_{v}\ \underbrace{bb \ldots b}_{w}$

① $a^{n+l} b^n \notin L$ ✓

② $a^n b^{n+l} \notin L$ ✓

③ $a \ldots abb \ldots aa \ldots b$ ✓ $\notin L$

K

$a^K b^K$

$2K+1 > K$

Game with Daemon :

Ex: $L = \{ 0^{n^2} \mid n \geq 0 \}$

k state DFA ✓

$y = 0^{k^2} \longrightarrow uvw$

$1 \leq |v| \leq k$, $|v| = l$

$\boxed{v = 0^{l}}$

$\boxed{uv^2w}$ $\xrightarrow{i=2}$ 0^{k^2+l}

$k^2 < \boxed{k^2+l} \leq k^2+k = k(k+1) < (k+1)^2$

Cannot be square quantity

$\alpha = \underbrace{aa\ldots a}_{n}\underbrace{bb\ldots b}_{n}$

$\alpha = x\ y\ z$

(k) DFA ✓

$y = uvw$

$v = a\ldots abb\ldots$

(i) choose $\longrightarrow uv^iw$ ✗

Def: L be a reg lang.

$\rightarrow \exists$ K ($\leftarrow$ Pumping Constant) state DFA

s.t. $\alpha = xyz$ $\quad |y| \geq K \longrightarrow y = uvw$

satisfying: (1) $|v| > 0$

(2) $|uv| \leq K$

(3) $xuv^iwy \in L$ $\quad \forall i \geq 0$

Proof: Do it Yourself

Instead of Use of P.L for proving closure properties of RL.

$a^n b^n$

$\boxed{abab}$ ✓

Ex: $\boxed{L} = \{ w \in \{a,b\}^* \mid \#a(w) = \#b(w) \}$

Suppose Yes (regular)

$\mathcal{L}(a^*b^*)$ is regular

$L \cap \mathcal{L}(a^*b^*) = \{a^n b^n \mid n \geq 0\}$

Reg

$\boxed{\neg \text{Regular}}$

Ex: $L = \{ a^m b^n \mid m \geq n \}$ — (1)

Suppose L regular ✓

$rev(L) = \{ b^n a^m \mid m \geq n \} \rightarrow$ is also regular

$L' = \{ a^n b^m \mid \underline{m \geq n} \} \rightarrow$ is also regular

(2) $L'' = \{ a^m b^n \mid n \geq m \} \rightarrow$ is also regular

$\boxed{\neg \text{Reg}}$

$L \cap L'' = \{ a^n b^n \mid n \geq 0 \}$

Exercises:

* $\boxed{L = \{ a^m b^n c^2 \mid m, n \geq 0, m \geq n \}}$

There are RL $\longrightarrow$ Some Languages are NOT Regular $\rightsquigarrow$?? (Next Lecture)

Context-Free Languages/Grammars:

Grammar $G = (N, \Sigma, P, S)$

nonterminal — terminal symbol — start NT

$P \subseteq N \times (N \cup \Sigma)^*$

$A \to \alpha$ $\Big\}$ $A \in N$, $\alpha \in (N \cup \Sigma)^*$

Derivation

$\alpha \xrightarrow[G]{1} \beta \to$ — -

$\alpha \xrightarrow[G]{*} \gamma$

{Sentential Form}

$S \xrightarrow[G]{+} \boxed{x}$ $\quad x \in \Sigma^*$

$L = \{ a^n b^n \mid n \geq 0 \}$

$w = \boxed{a^3 b^3}$ = a — b

aa — bb

$\boxed{aaa \mid bbb}$

ϵ

Derived

① $S \to aSb$
② $S \to \epsilon$

using

$aaSbb \xrightarrow[G]{1} aaaSbbb$

$S \xrightarrow[G]{*}$

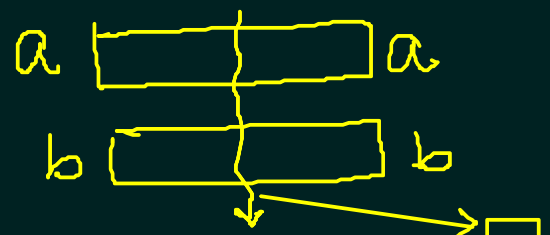
$S \to aSb \to aa\cancel{S}bb \to aabb \to$ sentence

Ex: $L = \{ w = w^{rev} \mid w \in \{a,b\}^* \}$

$$P \begin{cases} S \to aSa \mid bSb \\ S \to a \mid b \mid \epsilon \end{cases}$$

$G = (\{S\}, \{a,b\}, P, S)$

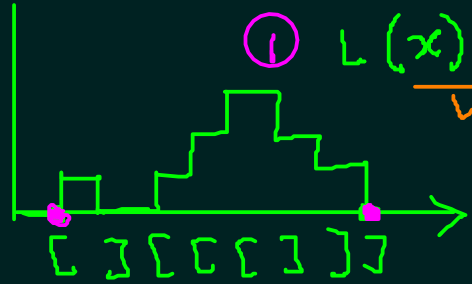
Rec:



Base: ϵ $\quad$ $a \mid b$

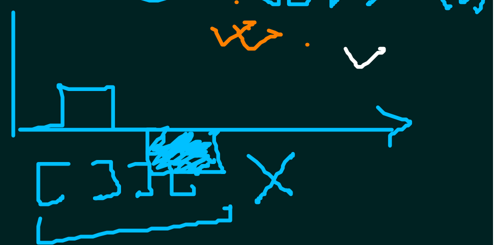
{Balanced Paranthesis}   [ ]   [[[ ]]][[ ]] ✓   [ ]] [✗   ( )( )( )

$$\boxed{S \to SS \mid [S] \mid \epsilon}$$



① $L(x) = R(x)$ ✓

② $L(y) \geq R(y)$ ✓



* $S \xrightarrow[G]{*} x$ is balanced.

iff ① & ② holds

"⇒" $S \xrightarrow[G]{*} x$ ⟶ Base : $S \xrightarrow[G]{0} S$ ⇒ ① & ② holds.

$\underbrace{S \xrightarrow[G]{n} \alpha}_{\text{hyp}} \xrightarrow[G]{1} \beta$

SS

[S] ⟶ [S → $L(y)+1 > R(y)$

[$\quad L(y)+1 > R(y)$

ϵ – $\quad$ [S] $\quad L(y)+1) \geq R(y)+1$

"⇐" <u>Do yourself</u> ⟶ if $\boxed{x} \in \Sigma^*$ ① & ② ⇒ $S \xrightarrow[G]{*} x$

Count → ✗

[{] } ✗

{ Count [ ] → ; Count { } → }

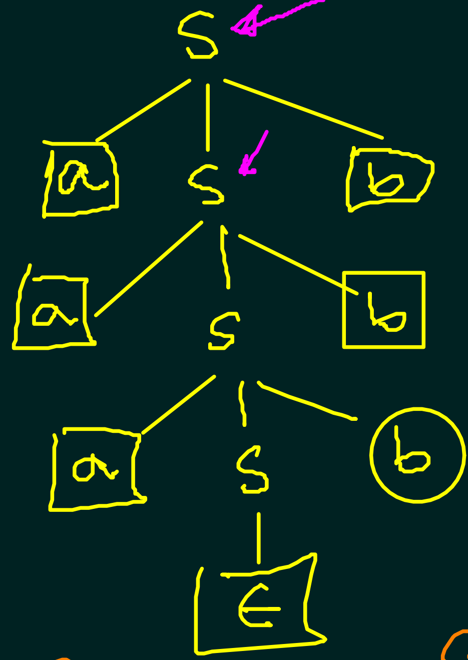
Stack → $\boxed{\text{PDA}}$

Derivation (Parse) Tree

$L = \{a^n b^n \mid n \geq 0\}$

$S \to aSb \mid \epsilon$

a^3b^3



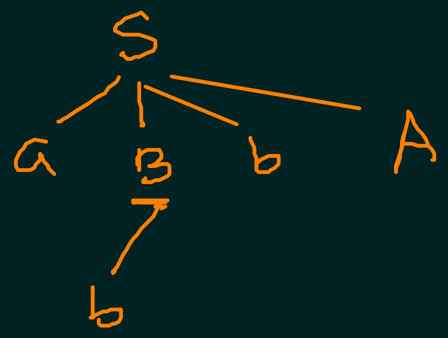
↳ Leftmost Derivation

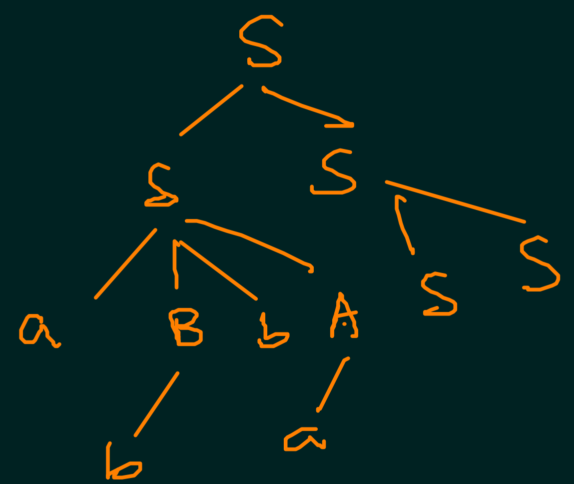
↳ Rightmost Derivation

$S \to aBbA \mid SS$

$A \to a$

$B \to b$





G1   G2

Ex: $S \to aAS \mid a$

$A \to SbA \mid SS \mid ba$

"aabbaa"

$S \to aAS \to aSbAS \to aabAS \to aabbaS \to aabbaa$

$S \to aAS \to aAa \to aSbAa \to aSbbaa \to aabbaa$

▷ Ambiguous Grammar { Two leftmost Derivation OR Two Rightmost Derivation

CFL ↔ Inherently Ambig

Normal form :

→ Chomsky NF → $A \to BC$, $A \to a$

→ Greibach NF → $A \to a B_1 B_2 \dots B_k$ $(k \geq 0)$

$\boxed{L(G) \neq \epsilon}$

$L(G) - \{\epsilon\} = L(G_{ch}) = L(G_{gr})$

$\boxed{S \to [S] \mid SS \mid \epsilon}$

Chomsky NF → $A \to \epsilon$ ✗ ✓

→ $A \to B$ ✗ ✓

$G(N, \Sigma, P, S)$

P : $A \to \alpha B \beta$, $\boxed{\cancel{B \to \epsilon}}$ ✗ - - - - → P' : $[A \to \alpha\beta$ ✓

→ $\boxed{\cancel{A \to B}}$ ✗ , $B \to \gamma$ - - - - → $A \to \gamma$ ✓

$(P \cup P')$

$G' = (N, \Sigma, P \cup P', S)$

$\boxed{L(G) \subseteq L(G')}$

$S \to \gamma\alpha B \beta \delta \Rightarrow \gamma\alpha\beta\delta \to x$

$L(G') \subseteq L(G)$

$S \to [S]$

$\cancel{S \to \epsilon}$ } $S \to [\,]$

" G_{ch} "

$S \to \cancel{[S]} \mid SS \mid \cancel{[\,]}$ AB

A B

$S \to ASB \mid SS \mid AB$ →

$\boxed{\begin{array}{l} A \to [\qquad C \to SB \\ B \to] \\ S \to AC \mid SS \mid AB \end{array}}$

Chomsky NF

$^{T_0}S \to {}^{T_1}A\, C^{T_3}$

$T_0 S \to {}^{T_0}S\, S^{T_0}$

$T_0 S \to {}^{T_1}A\, B^{T_2}$

$T_3 C \to {}^{T_0}S\, B^{T_2}$

$T_1 A \to [$

$T_2 B \to]$

$T_3 \to T_0 T_2$ $\Rightarrow$ $T_3 \to T_1 T_3 T_2 \mid T_0 T_0 T_2 \mid$

$T_1 T_2 T_2$

$A \to a\alpha$

$A \to \boxed{a} B_1 \dots B_k$

$T_0 \to [T_3$

$T_0 \to T_0 T_0$

$T_0 \to [T_2$

$T_3 \to [T_3 T_2 \mid T_0 T_0 T_2 \mid$

$[T_2 T_2$

$T_0 \to [T_3 T_0 T_0 \mid [T_2 T_0 T_0$

$T_3 \to T_0 T_0 T_2 T_2 E \mid$

$[T_2 T_2 T_2 E$

$T_0 T_0 T_0$

$T_0 \boxed{T_0 T_0 T_0}$

A_3

$A_3 \to A_3 \boxed{A_1 A_3 A_2} \mid b A_3 A_2 \mid a$

$A_3 \to \boxed{A_3} \boxed{A_1 A_3 A_2}$

$\boxed{A_1 A_3 A_2}$

$T_0 \to [T_3 \mid [T_2 \mid$

$[T_3 X \mid [T_2 X \mid$

$A_3 \to b A_3 A_2 \boxed{A_1 \cancel{A_3} A_2} \mid a\, A_1 A_3 A_2 \mid b A_3 A_2 \mid a$

B_3

$X \to [T_3 \mid [T_2 \mid$

$[T_0 X \mid [T_3 X$

$[T_2 X \mid [T_3 X X \mid [T_2 X X$

$B_3 \to A_1 A_3 A_2 \mid A_1 A_3 A_2 B_3$

$a\alpha$