SUMMARY

Deterministic Finite Automaton (DFA):

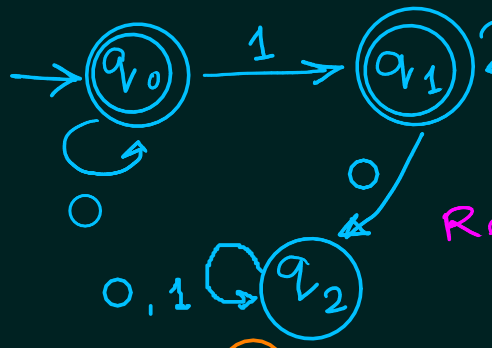
↳ $\langle Q, \Sigma, q_0, \delta, F \rangle$

set of states → Q; alphabet → Σ; start state → q_0; Transition function → δ; Final states → F

$q_0 \in Q$

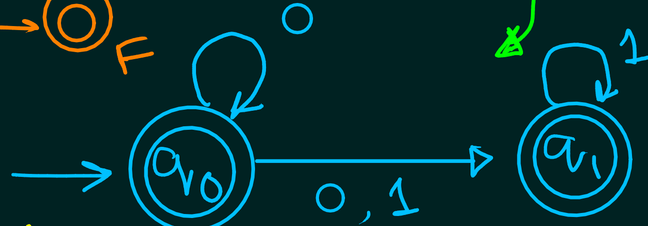
$F \subseteq Q$

$\delta : Q \times \Sigma \to Q$



Accepts Regular Lang. $0^* 1^*$

Accept Condition: $w \in L$ if $\hat{\delta}(q_0, w) = q_f \in F$



Non-deterministic Finite Automata (NFA):

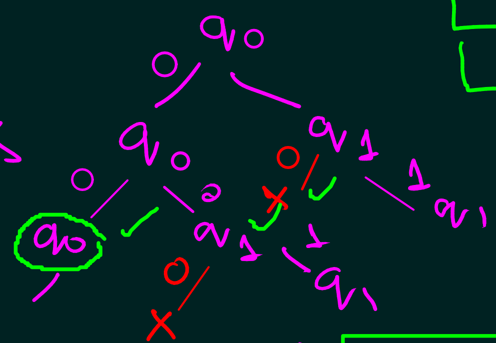
↳ $\langle Q, \Sigma, q_0, \delta, F \rangle$ where $\delta : Q \times \Sigma \to 2^Q$ (Power states of Q)

↳ no transition

↳ multiple transition

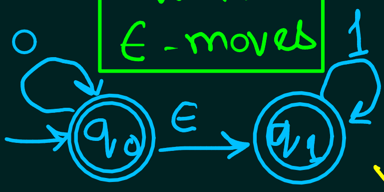
Accept condition: $w \in L$ if $\hat{\delta}(q_0, w) = \{q_1, q_2, \dots, q_n\}$ and $\exists q_i \in F$

at least one path leads to final states



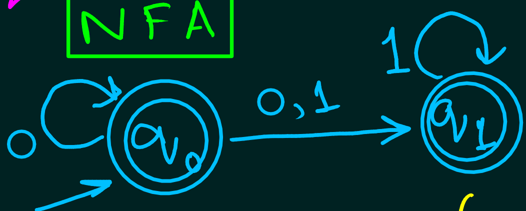
$\boxed{\text{Regular Expression}}$ $\xrightarrow{1}$ $\boxed{\text{NFA with } \epsilon\text{-moves}}$ $\xrightarrow{2}$ $\boxed{\text{NFA}}$ $\xrightarrow{3}$ $\boxed{\text{DFA}}$ $\xrightarrow{4}$ $\boxed{\text{Minimized DFA}}$ $\xrightarrow{5}$ Regular Expression

$0^* 1^*$



→ $\epsilon\text{-closure}(q_0) = \{q_0, q_1\}$

$[\, 0^*\epsilon = 0^*, \quad \epsilon 1^* = 1^* \,]$



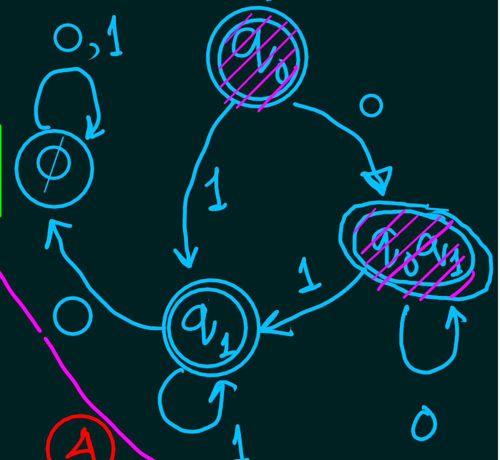
$\delta(Q, a) = \bigcup_{q \in Q} \delta(q, a)$

$\delta(q_0, 0) = \{q_0, q_1\}$

$\delta(q_0, 1) = \{q_1\}$

$\delta(\{q_0, q_1\}, 0) = \{q_0, q_1\}$

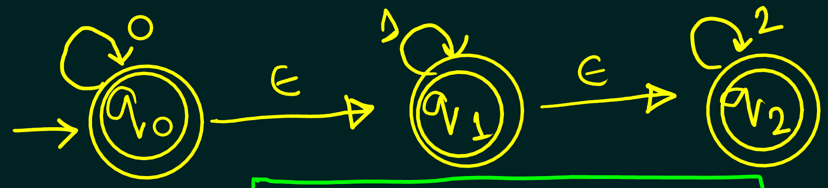
$\delta(\{q_0, q_1\}, 1) = \{q_1\}$



$q_0 = q_0 q_1$

$0^* 1^* 2^*$

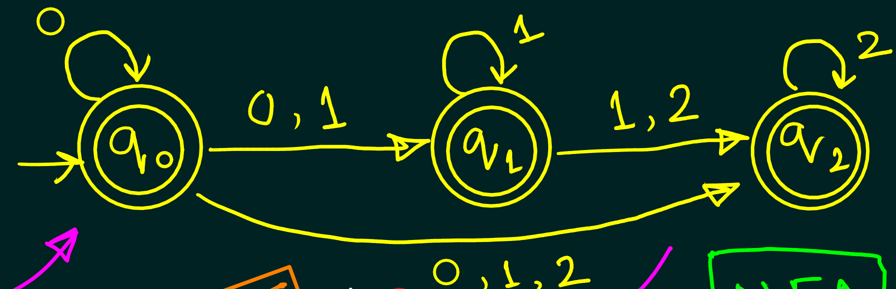
Regular Expression



1 ?

NFA with ε moves

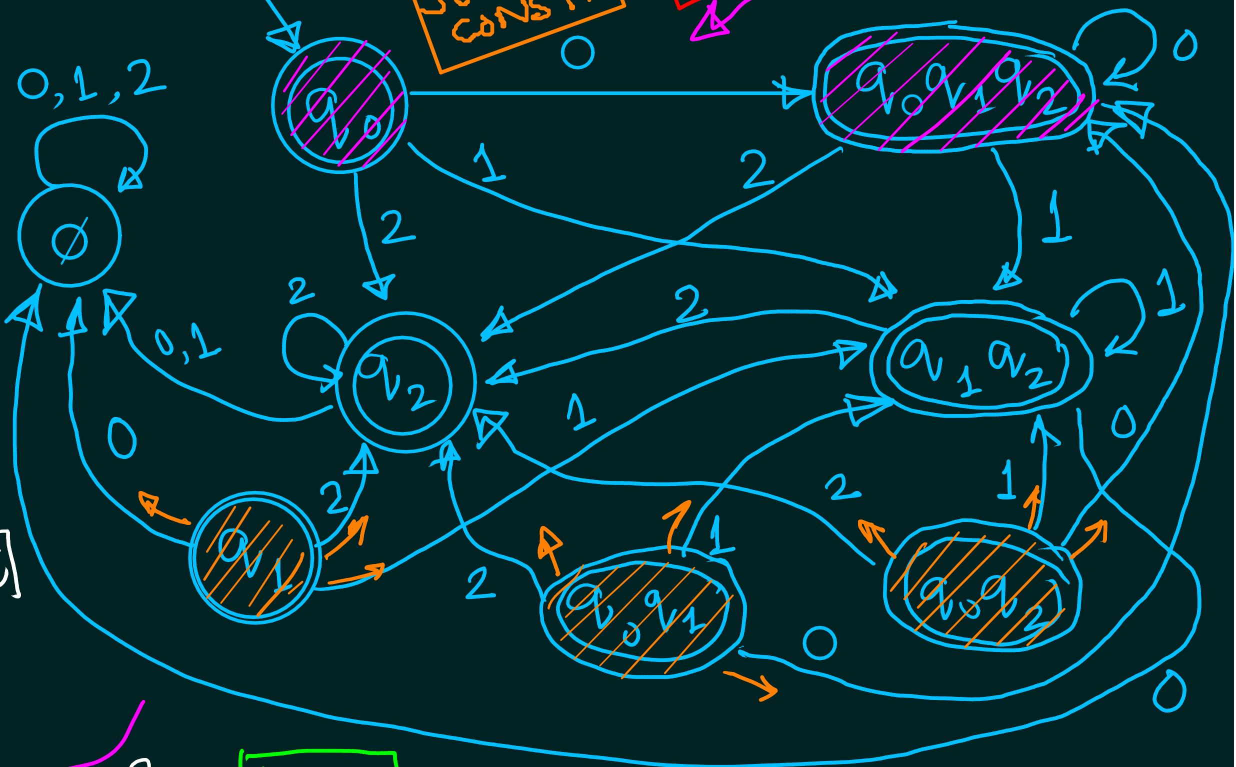
2



NFA

SUBSET CONST.

3



▶ Redundant States [unreachable from q_0] } $[q_0 q_2]$, $[q_0 q_1]$, $[q_1]$

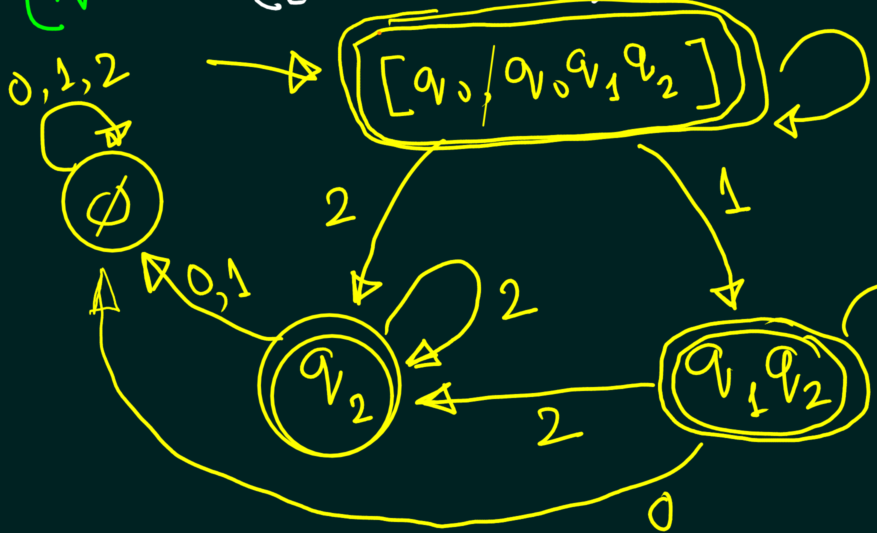
▶ Equivalent states:

$[q_0] \equiv [q_0 q_1 q_2]$

Why: $\hat{\delta}([q_0], w) = \hat{\delta}([q_0 q_1 q_2], w)$

OR $\delta([q_0], a) = [p]$

$(\forall a)\ \delta([q_0, q_1, q_2], a) = [q]$ } $\Rightarrow [p] \equiv [q]$



4 ?

DFA

Minimized DFA

Unique (Myhill-Narode Thm)

Regular Expression

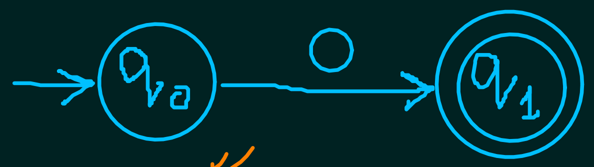
5 ?

Other Closure Properties.

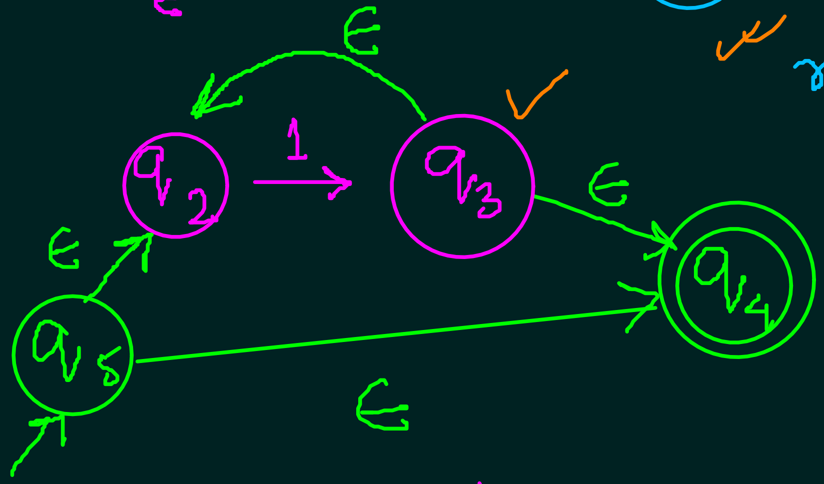
TODAY'S LECTURE

1 Regular Expression → NFA with ϵ-Moves → NFA

$(0\,1^* + 1)$



$r = 0$



$r = 1^*$

$(0\,1^* + 1) = \{0\,\boxed{1}, 01, 011, 0111, 01111 \ldots, \ldots\}$

RE   RL   L

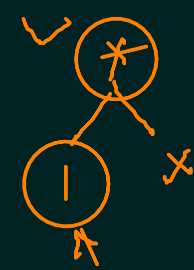
$\boxed{w \in L}$?

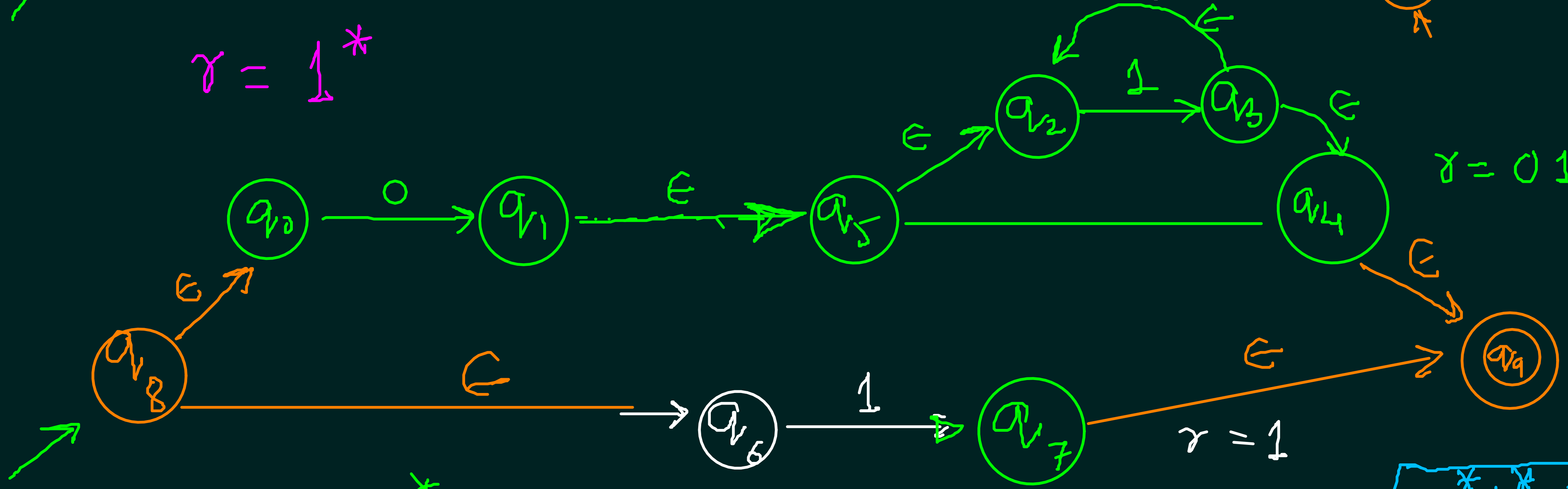
$(01^* + 1) + 0(11)^*$

$\boxed{1}^* = \epsilon + 1^+$

$= \epsilon + \epsilon\,1\,\epsilon$

$+ \epsilon\,(1\epsilon)^*\,\epsilon$





$r = 0\,1^+$

$r = 1$

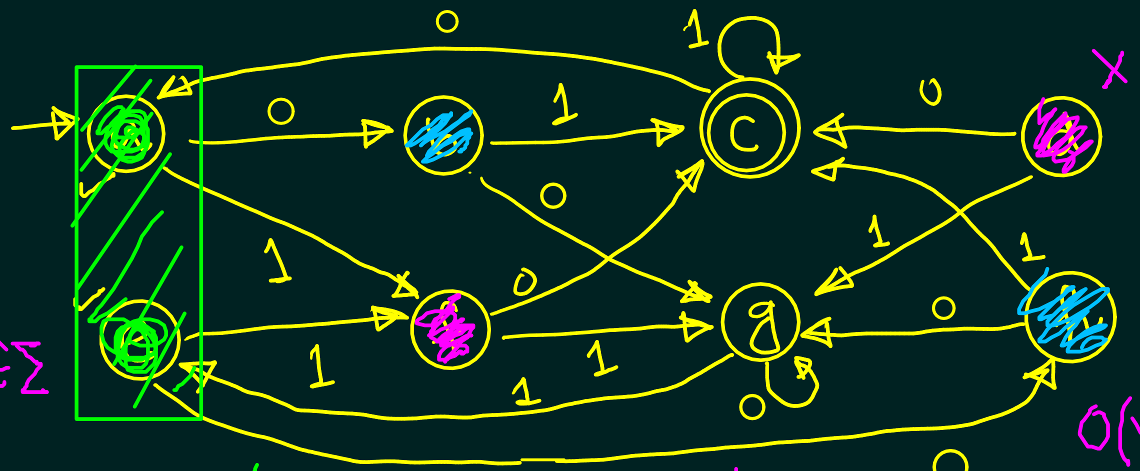
$r = 0\,1^* + 1$

$\boxed{0^*1^*2^*}$

DFA → Minimized DFA

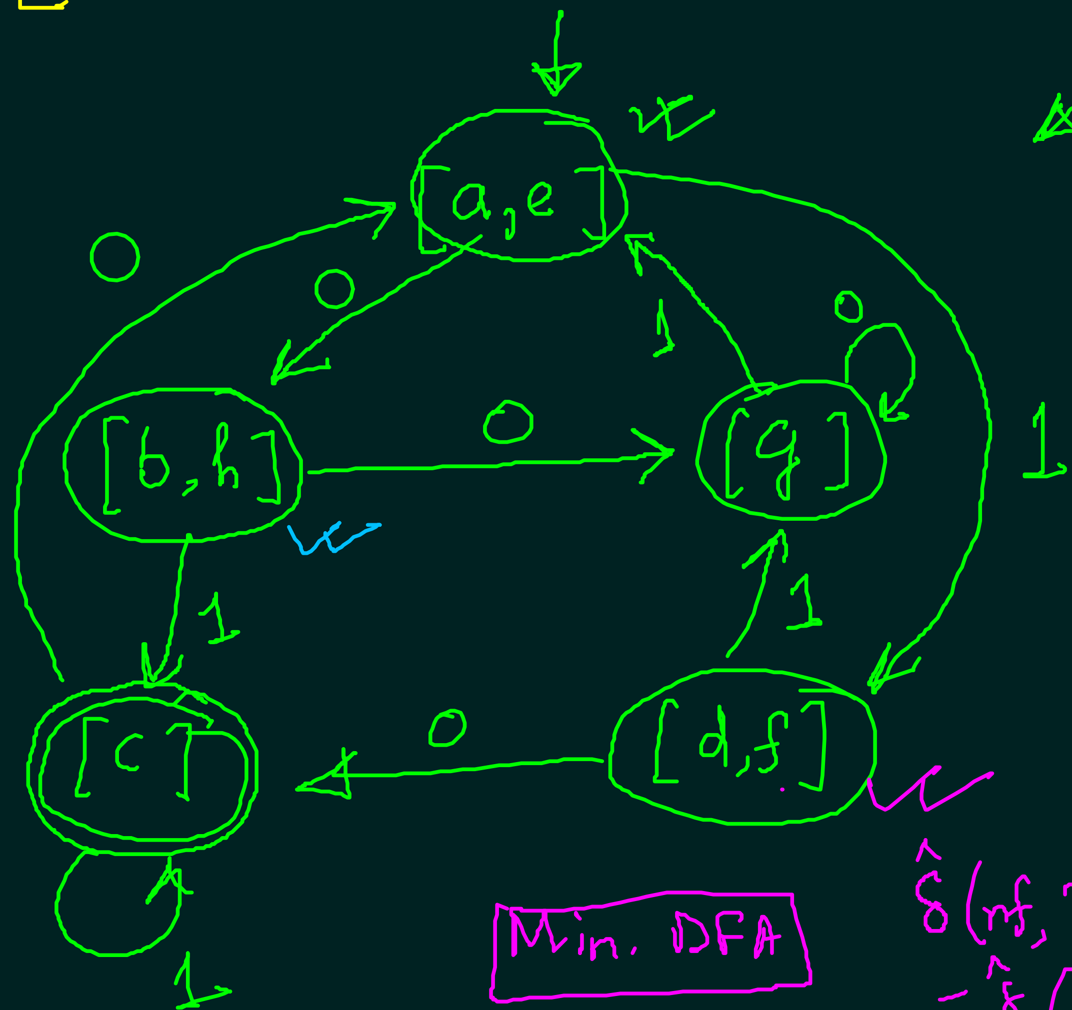
$[p] \equiv [q]$ iff

if $[\delta([p], a)] = [\delta([q], a)], \forall a \in \Sigma$



$O(V+E)$

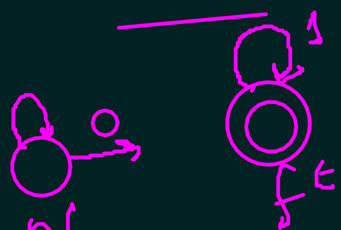
$+ \; k'(n + 2n)$



Min. DFA

* Remove RED.

* Merge Equi.



	a	b	c	d	e	f	g
a							
b	X						
c	X	X					
d	X	X	X				
e		X	X	X			
f	X	X	X		X		
g	X	X	X	X	X	X	
h	X		X	X	X	X	X

$O(n^2)$

$\hat{\delta}(nf, w) = \hat{\delta}(f, w)$

$O\left[kn^2 - k(n-1)^2\right]$

Myhill-Narode Th. :

$\boxed{x \, R \, y \quad \text{iff} \quad \delta(q_0, x) = \delta(q_0, y)}$ → Equiv. ✓

$\{ x_1, x_2 \, - - - - \}$

$\{ [x] , [x'] , [x''] \, - - - - \}$

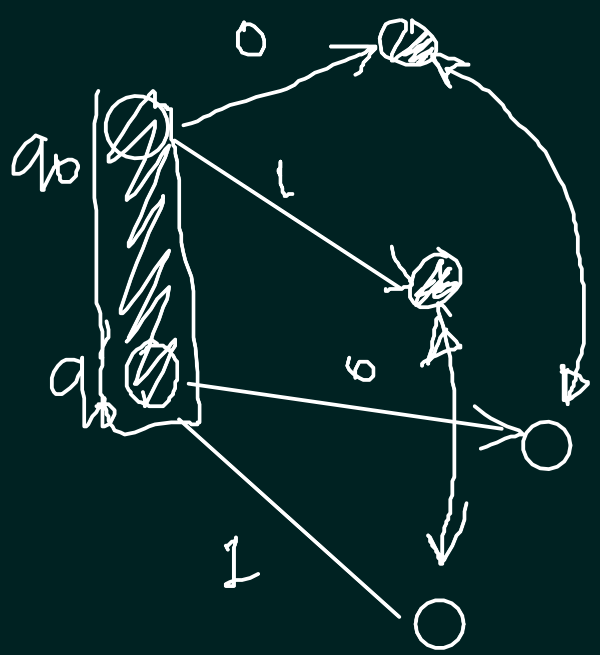
→ algeb. mb of min.

↓

"finite index"

$q_0 \to q_f$, q_f

$\overline{0^n 1^n}$



$[0w] \equiv [0w]$

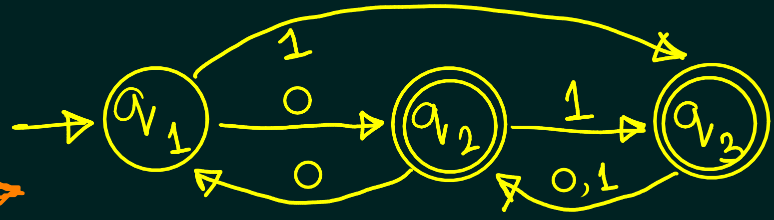
quotient Set Cnt.

$$\left[\begin{array}{ll} z \in \Sigma^* & x R y \\ xz \in L \Rightarrow & yz \in L \end{array} \right]$$

DFA → Regular Expression

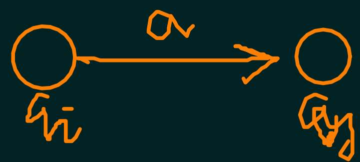
r_{ij}^{k} → $R = \underbrace{r_{12}^{3}} + \underbrace{r_{13}^{3}}$

going thru



Min DFA

$$\boxed{r_{ij}^{0}} = \begin{cases} \underline{a \mid \delta(q_i,a)=q_j,\ i \neq j} \\ \{a \mid \delta(q_i,a)=q_j\} \cup \{\epsilon\},\ i=j \end{cases}$$



→k

$$r_{ij}^{K} = \underbrace{r_{ik}^{K-1}\left(r_{KK}^{K-1}\right)^{*} r_{Kj}^{K-1}}_{K^{th}} + r_{ij}^{K-1}$$

$\not k^{th}$

$$r_{22}^{K=1} = r_{21}^{0}\left(r_{11}^{0}\right)^{*}\left(r_{12}^{0}\right) + r_{22}^{0}$$

$$= 0\ \underline{\epsilon^{*}\ 0} + \underline{\epsilon}$$

$r_{32}^{0} = 0+1$ $\qquad$ $r_{11}^{0} = \epsilon$

	K=0	K=1	K=2
r_{11}^{K}	$\boxed{\epsilon}$	ϵ	$(00)^*$
r_{12}^{K}	0	0	$0(00)^*$
r_{13}^{K}	1	1	0^*1^*
r_{21}^{K}	0	0	$0(00)^*$
r_{22}^{K}	ϵ	$\boxed{\epsilon+00}$	$(00)^*$
r_{23}^{K}	1	$1+01$	0^*1
r_{31}^{K}	$\varnothing$	$\varnothing$	$(0+1)(00)^*0$
r_{32}^{K}	$\boxed{0+1}$	$0+1$	$(0+1)(00)^*$
r_{33}^{K}	ϵ	ϵ	$\epsilon+(0+1)0^*1$