

Propositional Logic:

- ↳ Constant (T, F)
- ↳ Boolean variables
- ↳ Connectives ($\wedge, \vee, \neg, \rightarrow$)

SUMMARY

Boolean Formula Encoding

- ① Identify variables for simple statement / arguments.
- ② Apply connectives to form Boolean Formula

Deduction Procedure:

↳ Truth Table checking (exponential w.r.t. vars.)

↳ Inference Rules

$\frac{p \quad p \rightarrow q}{\therefore q}$	$\frac{\neg q \quad p \rightarrow q}{\therefore \neg p}$	$\frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$	$\frac{p \vee q \quad \neg p}{\therefore q}$
Modus Ponens	Modus Tollens	Syllogism	Disjunctive Syllogism

$\frac{p \wedge q \quad p \rightarrow (q \rightarrow r)}{\therefore r}$	$\frac{p \rightarrow q \quad r \rightarrow s \quad p \vee r}{\therefore q \vee s}$	$\frac{p \rightarrow q \quad r \rightarrow s \quad \neg q \vee \neg s}{\therefore \neg p \vee \neg r}$	ETC....
Conditional Proof	Constructive Dilemma	Destructive Dilemma	

Absorption Rules $\rightarrow a \vee (a \wedge b) \equiv a$
 $\rightarrow a \wedge (a \vee b) \equiv a$

Rules are applied to reduce # vars

$$F_1 \wedge F_2 \wedge \dots \wedge F_n \rightarrow G$$

(Valid) \leftarrow Tautology \equiv All interpretations results TRUE

(falsify) \rightarrow Unsatisfiable \equiv All interpret. results FALSE

▷ Satisfiable: At least one interpret. results TRUE

▷ Invalid: At least one interpret. results FALSE.

Predicate Logic (First-Order):

↳ Boolean variables extended to Boolean Predicates

↳ Constants extended to Functions.

↳ Quantifiers (\forall, \exists)

Today's Lecture

No contractors are dependable. Some engineers are contractors.
 Therefore, some engineers are not dependable.

$$F_1 \forall x [\text{cont}(x) \rightarrow \neg \text{dep}(x)]$$

$$F_2 \exists x [\text{enggr}(x) \wedge \text{cont}(x)]$$

$$G \exists x [\text{enggr}(x) \wedge \neg \text{dep}(x)]$$

$$\exists x P(x) \equiv \neg \forall x [\neg P(x)]$$

$$\neg \forall x [\text{enggr}(x) \rightarrow \neg \text{cont}(x)] \equiv \exists x [\text{enggr}(x) \wedge \text{cont}(x)]$$

$$\neg \forall x [\text{cont}(x) \rightarrow \neg \text{enggr}(x)]$$

$$\checkmark T \equiv (F_1 \wedge F_2 \rightarrow G) \text{ or } (F_1 \wedge F_2 \wedge \neg G) \text{ is unsat}$$

Every passenger either travels in first class or second class. Each passenger is in second class if and only if he or she is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore, some passengers travel in second class.

$$F_1 \forall x [\text{pass}(x) \rightarrow (\text{first}(x) \vee \text{sec}(x))] \quad (\text{exclusive}) \quad (\neg f(x) \wedge \neg s(x)) \vee (\neg f(x) \wedge s(x))$$

$$F_2 \forall x [\text{pass}(x) \rightarrow (s(x) \leftrightarrow \neg w(x))] \equiv \forall x [\text{pass}(x) \wedge s(x) \rightarrow \neg w(x)] \vee \forall x [\text{pass}(x) \wedge \neg w(x) \rightarrow s(x)]$$

$$F_3 \exists x [\text{pass}(x) \wedge w(x)]$$

$$G \exists x [\text{pass}(x) \wedge s(x)]$$

$$F_4 \neg \forall x [\text{pass}(x) \rightarrow w(x)] \equiv \exists x [\text{pass}(x) \wedge \neg w(x)]$$

Everyone likes everyone.

$$\forall x \forall y [like(x,y)] \checkmark$$

$$A \rightarrow S_1$$

Someone likes someone.

$$\exists x \exists y [like(x,y)] -$$

$$B \rightarrow S_2$$

Everyone likes someone.

$$\forall x \exists y [like(x,y)] \checkmark$$

Someone likes everyone.

$$\exists x \forall y [like(x,y)] \checkmark$$

not equivalent

Everyone is liked by everyone.

$$\forall y \forall x [like(x,y)] \checkmark$$

Someone is liked by someone.

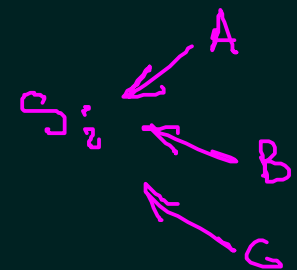
$$\exists y \exists x [like(x,y)] -$$

Someone is liked by everyone.

$$\exists y \forall x [like(x,y)] \checkmark$$

Everyone is liked by someone.

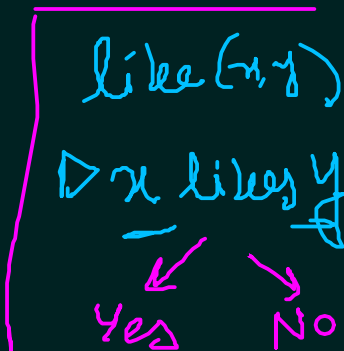
$$\forall y \exists x [like(x,y)] \checkmark$$



If everyone likes everyone, then someone likes everyone.

← Tautology

$$\forall x \forall y [likes(x,y)] \rightarrow \exists x \forall y (likes(x,y))$$



If some person is liked by everyone, then that person likes himself/herself.

$$\exists x \forall y (likes(y,x)) \rightarrow likes(x,x)$$

← Tautology

If x is greater than y and y is greater than z , then x is greater than z .

$$\forall x \forall y \forall z [gt(x, y) \wedge gt(y, z) \rightarrow gt(x, z)] \quad \text{---ve}$$

The age of a person is more than the age of his/her child.

Fund Symb. $Age(x)$ $gt(x, y)$ $Age(y)$ $A\ child(y, x)$

$$\forall x \forall y [child(y, x) \rightarrow gt(Age(x), Age(y))]$$

The age of a person is greater than the age of his/her grandchild.

$$\forall x \forall y \forall z [child(y, x) \wedge child(z, y) \rightarrow gt(Age(x), Age(z))]$$

$gc(x, z)$

$a \rightarrow b$
 FALSE
 "vacuous"

The sum of ages of two children is never more than or equal to the sum of ages of their parents.

— H/W —

$\forall x \text{ Pred}(x, y)$
 bound \rightarrow free

$\forall x [P(x, y) \wedge \exists z Q(x, y, z, w)]$
 free \rightarrow free \rightarrow free

(Scope rule)

$\forall x [P(x)] \vee Q(y)$
 free

$\forall x [P(x, y) \wedge \exists y \exists z Q(x, y, z, w)]$
 free \rightarrow free \rightarrow free

Const: $T, \perp, \text{value}, 12, 104, M, N,$

Var: $x, y,$

Pred: $\text{pred}(x, y), Q(y, z, w)$

Funct.: $\text{Age}(x), F(x, y, z)$

Terms: Const \checkmark , Var \checkmark

$F(t_1, \dots, t_k)$ is a term.

Formula:

prop is Formula $\begin{matrix} \uparrow \\ \perp \end{matrix}$
 $\text{pred}(t_1, \dots, t_k)$ Formula

$f_1 \wedge f_2, f_1 \vee f_2, f_1 \rightarrow f_2, \neg f_1$ formulas

$\text{gt}(\text{Age}(w), 12)$

$\text{gt}(a, b)$

Def.
 (Recursive)

$\text{Sum}(\frac{\text{Age}(x)}{t_1}, \frac{\text{Age}(y)}{t_2})$

$\forall x_1 \dots \forall x_n \text{ pred}(x_1, \dots, x_n)$

$\exists x_1 \dots \exists x_n \text{ pred}(x_1, \dots, x_n)$
 formulas

Any computable function can be expressed using P.L.
(argument)

$x \rightarrow \boxed{P} \rightarrow y \quad \forall x \exists y \{x\} \cap P \rightarrow \{y\} \leftarrow \underline{\text{deduce}}$

$\forall \text{pred } \neg \text{th}(\text{pred}(x)) \times$
 $\forall \text{Age } \neg x(\text{Age}(x)) \times$ } limitations

Higher order logic

Example

$\forall p \left[(p(0) \wedge (\forall x (p(x) \rightarrow p(S(x)))) \rightarrow \forall y [p(y)] \right]$

— What is expressed?

→ Russell's Paradox ✓

→ Liar's Paradox

undecidable

→ Next class:
— deduction
— limitations