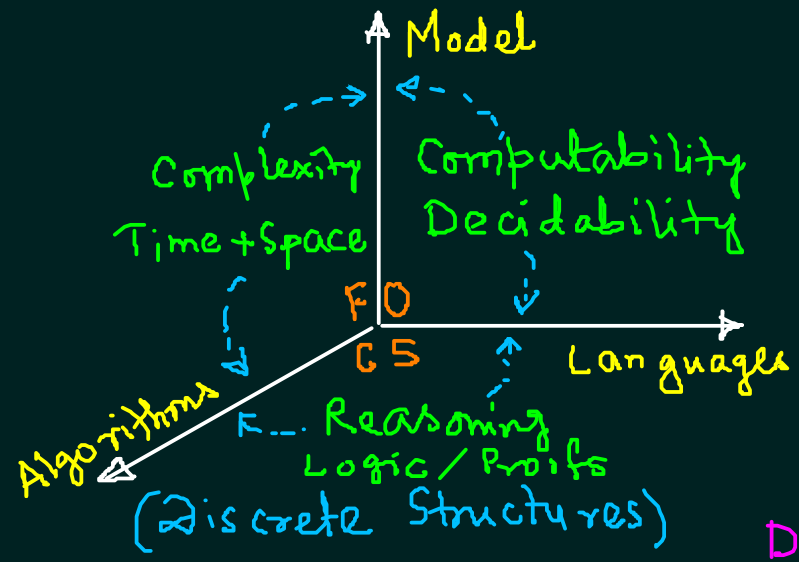
SUMMARY

▷ Landscape of FOCS:



Computational Problems

$I \rightarrow O$

Set of inputs → I; set of soln. → O

⇓

language of problem $(\mathcal{L})$

↙ Decision ↓ Function ↘ optimization

Computational Solution

$f : I \rightarrow O$

↖ compute

⇓

Input $x \in I$

$x \in \mathcal{L}$.

(language containment)

↙ Solvable ↘ Unsolvable

▷ By Computation, we CANNOT solve all problems. Ex: Halting Prob.

▷ Proof Techniques:

① Direct and Indirect

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

② By Contradiction

$p \rightarrow q$ and $p \xrightarrow{\text{derive}} \neg q$

③ Existance Proof.

$\exists x\, P(x)$ or $\forall x \exists y\, P(x,y)$

↙ Constructive ↘ Non-Construct

④ Proof by Cases

$(p_1 \vee p_2) \rightarrow q \equiv (p_1 \rightarrow q) \wedge (p_2 \rightarrow q)$

⑤ Proof by disjunction

$p \rightarrow q \vee r$ → $p \wedge \neg q \rightarrow r$; → $p \wedge \neg r \rightarrow q$

Disprove:

$\exists x\, \neg P(x)$

one counter Ex.

⑥ Cycle of Implications

$p_1 \rightarrow p_2 \rightarrow p_3$ (cycle)

⑦ Proof by Induction

$P_1, P_2, \ldots, P_n$ true

$\therefore P_{n+1}$ is true.

→ Strong
→ Weak

LOGIC ▷ ▷ ▷

[next lectures]

If two meetings are happening parallelly then I am confused.

A_1 (T / F) $\longrightarrow$ A_2

If it is raining then I must carry umbrella.

A_1 (T / F) $\longrightarrow$ A_2 (Connective)

$A_1 \rightarrow A_2 \equiv \neg A_1 \vee A_2$

(1) Variables, Constant (T, ⊥)

(2) Connectives $\wedge$, $\vee$, $\neg$, $\rightarrow$, $\leftrightarrow$ (if and only if)

$A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$

Syntax: Var, Const are Prop. Logic Formula

Semantics: p, q are PL formulas then $(p \wedge q)$, $(p \vee q)$, $\neg p$, $(p \rightarrow q)$.

Truth Table

a	b	$a \rightarrow b$
T	T	T
T	F	F
F	T	T
F	F	T

Interpretation

$(a \vee \neg a)$

Satisfiable → at least one int. where f true

(Tautology) Valid → all interp. where f true

invalid

(unsatisfiable)

If I am the VP of Gymkhana, then I am well-known in IIT.
I am the VP of Gymkhana. Therefore, I am well-known in IIT.

$F_1: v \to w$
$F_2: v$ $\quad G: w$
$(F_1 \wedge F_2) \to G$?

If Devika is the VP of Gymkhana, then Devika is well-known in IIT.
Devika is the VP of Gymkhana. Therefore, Devika is well-known in IIT.

If Neha is the VP of Gymkhana, then Neha is well-known in IIT.
Neha is NOT the VP of Gymkhana. Therefore, Neha is NOT well-known in IIT.

$F_1: v \to w$
$F_2: \neg v$
$G: \neg w$

v	w	F_1	F_2	G	$F_1 \wedge F_2$	G	$(F_1 \wedge F_2) \to G$
T	T	T	T	T	T	T	T
T	F	F	T	F	F	F	T
F	T	T	F	T	F	T	T
F	F	T	F	F	F	F	T

Tautology

T	T	T	F	F	F	F	T
T	F	F	F	T	F	T	T
F	T	T	T	F	T	F	F
F	F	T	T	T	T	T	T

sat ✓
valid ✗
invalid ✓
unsat ✗

Deduction :

⓪ $p \wedge (p \vee q) \equiv p$ ✓

$p \vee (p \wedge q) \equiv p$

} absorbtion

① $p \to q$, p

$\therefore q$ ✓

(Modus Ponens)

$(p \to q) \wedge p \equiv (\neg p \vee q) \wedge p$

$\equiv (\neg p \wedge p) \vee (q \wedge p)$

$\equiv q \wedge p \Rightarrow q$

True

② $p \to q = T$, $\neg q = T$

$\therefore \neg p = T$

(Modus Tollens)

③ $p \to q$, $q \to r$

$\therefore p \to r$ ↓

④ $p \to q$, $r \to s$, $p \vee r$

$\therefore q \vee s$ ↓

⑤ $p \to q \vee r$, $p \wedge \neg q$

$\therefore r$

De Morgan :

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

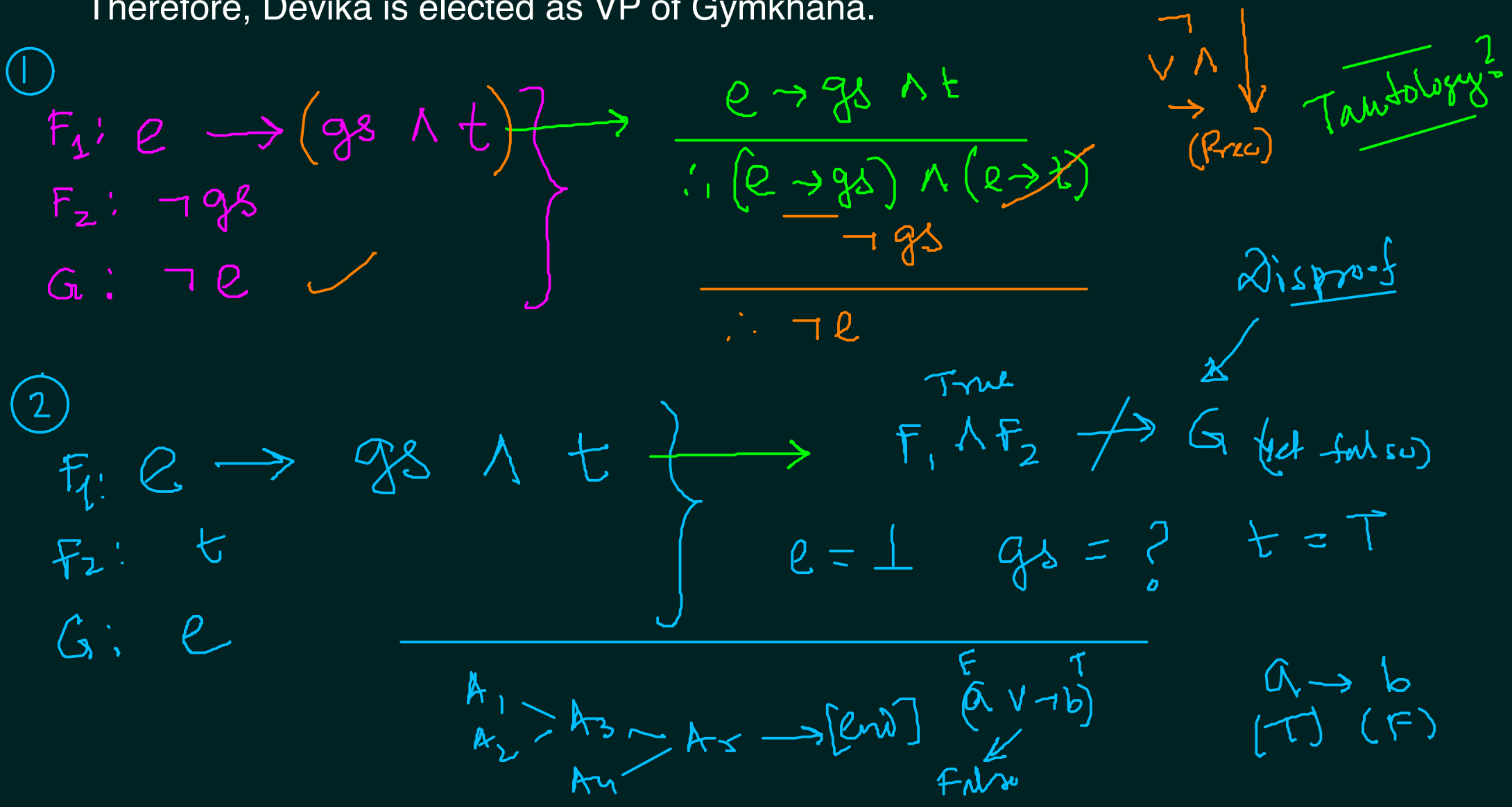
Deduction Rules

Simplify ✓

Reduce search space

If Devika is elected as the VP of Gymkhana, then Akshay is chosen as a G-Sec AND Azhar is chosen as a Treasurer. Akshay is NOT chosen as a G-Sec.
Therefore, Devika is NOT elected as VP of Gymkhana.

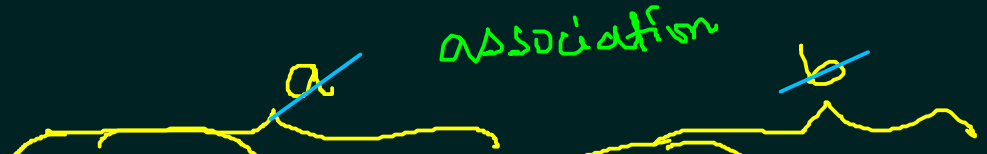
If Devika is elected as the VP of Gymkhana, then Akshay is chosen as a G-Sec AND Azhar is chosen as a Treasurer. Azhar is chosen as a Treasurer.
Therefore, Devika is elected as VP of Gymkhana.



If Ninaad is elected as the VP, then Ayushi is chosen as a G-Sec OR
Devang is chosen as a Treasurer. Ayushi is NOT chosen as a G-Sec.
Therefore, if Ninaad is elected as VP then Devang is chosen as a Treasurer.

If Ninaad is elected as the VP, then EITHER Ayushi is chosen as a G-Sec OR
Devang is chosen as a Treasurer, but not both. Ayushi is NOT chosen as a G-Sec.
Therefore, if Ninaad is elected as VP then Devang is chosen as a Treasurer.

— Do these yourself —



Predicate Logic

Quantifiers

some $\exists$
all $\forall$

Wherever Ankush goes, so does the pet dog. Ankush goes to school. So, the dog goes to school.

No contractors are dependable. Some engineers are contractors. Therefore, some engineers are not dependable.

All actresses are graceful. Anushka is a dancer. Anushka is an actress. Therefore, some dancers are graceful.

HW

Predicates

goes(x, y);
x goes to y

Boolean

G: goes(Dog, School)

F_1: $\forall x$ (goes(Ankush, x) $\rightarrow$ goes(Dog, x))

F_2: goes(Ankush, School)

$F_1 \wedge F_2 \rightarrow G$

HW

Every passenger either travels in first class or second class. Each passenger is in second class if and only if he or she is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore, some passengers travel in second class.

$a \rightarrow \neg b$

$\forall x (cont(x) \rightarrow \neg dep(x)) \equiv \neg \exists x (cont(x) \wedge dep(x))$

$\exists x (eng(x) \wedge cont(x)) \equiv \neg [\forall x (eng(x) \rightarrow \neg cont(x))]$

$\exists x\ P(x) \equiv \neg \forall (x)\ \neg P(x)$ Duality