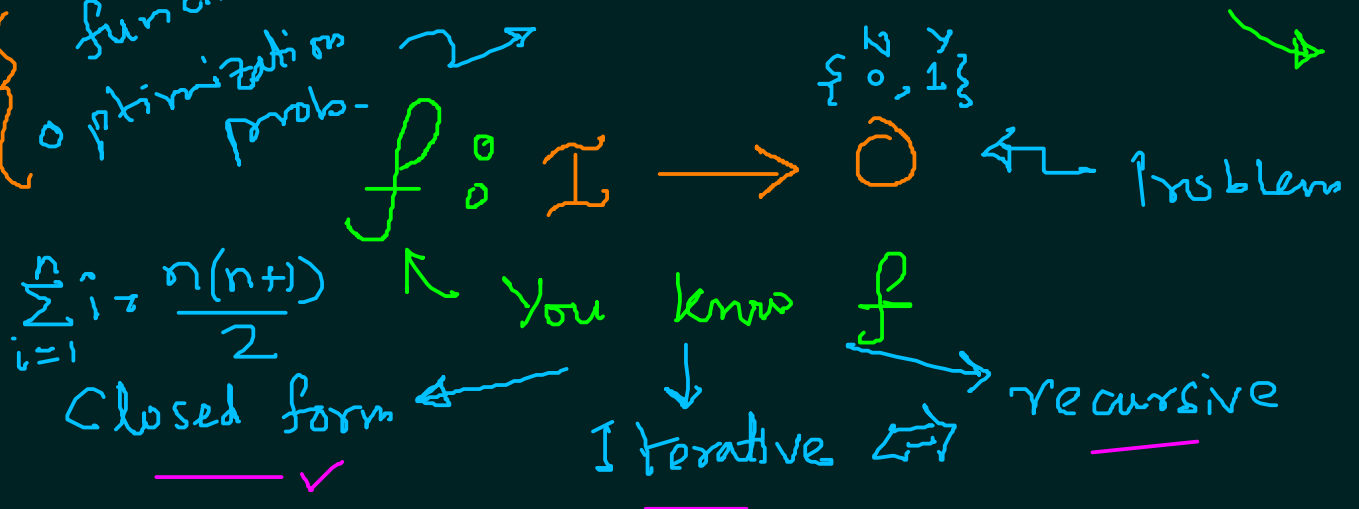


# FOUNDATIONS OF COMPUTING SCIENCE (FOCS)

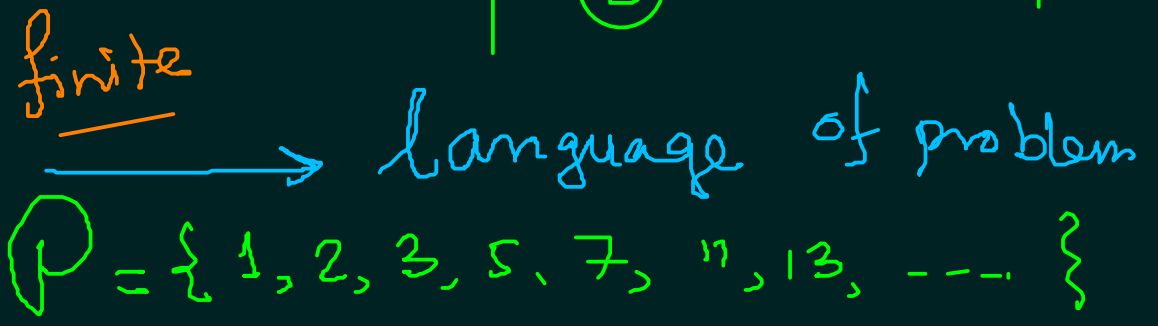
- Computational Problems
- ① Find whether  $n \in \mathbb{Z}^+$  is prime? → Yes  
→ No
  - ② Find factors of  $n \in \mathbb{Z}^+$ ? → { , , } set/result
  - ③ Find shortest route from Airport to Victoria in Kolkata? → minimization
- Decision problem
- function prob.
- optimization prob.



- Computing Solution?
- ① primality testing
  - ② integer factorization
  - ③ shortest path

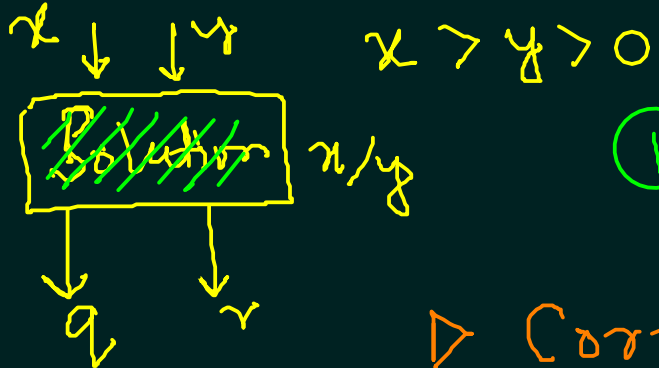
## Represent Problems

- ↳ English Description
- ↳ Math Formulation



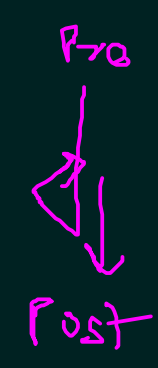
$n \in P$ ?

language containment / Inclusion problem



①  $\begin{cases} q \leftarrow x/y \\ r \leftarrow x - q \cdot y \end{cases}$

②  $\begin{cases} q \leftarrow 0 \\ \text{while } (x > 0) \\ \quad x \leftarrow x - y \\ \quad q \leftarrow q + 1 \\ \text{end while} \\ r \leftarrow x + y \end{cases}$



▷ Correct?

$\forall x \forall y \exists q \exists r (x > y > 0 \rightarrow x == q \cdot y + r \ \& \ r < y)$

forall      there exist      Logic (Predicate / 1st order)

$\{Pre\} \ \& \ \{Prog\} \rightarrow \{Post\}$  Hoare Logic (Triplet)

→ Are there unsolvable problems?

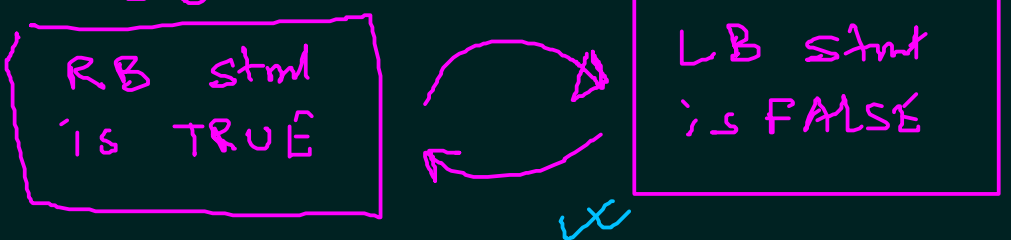
1900 - David Hilbert (23 problems for Century)

3rd → Is Math Logic Axioms are

1923 - David Hilbert ("We must know we should know")

Consistent & Complete?

1923 - Kurt Gödel



Are there unsolvable problems?

Yes  $\leftarrow$  ①  $\dashrightarrow$

$\hookrightarrow$  There are problems which are not computable.

$\hookrightarrow$  How many?

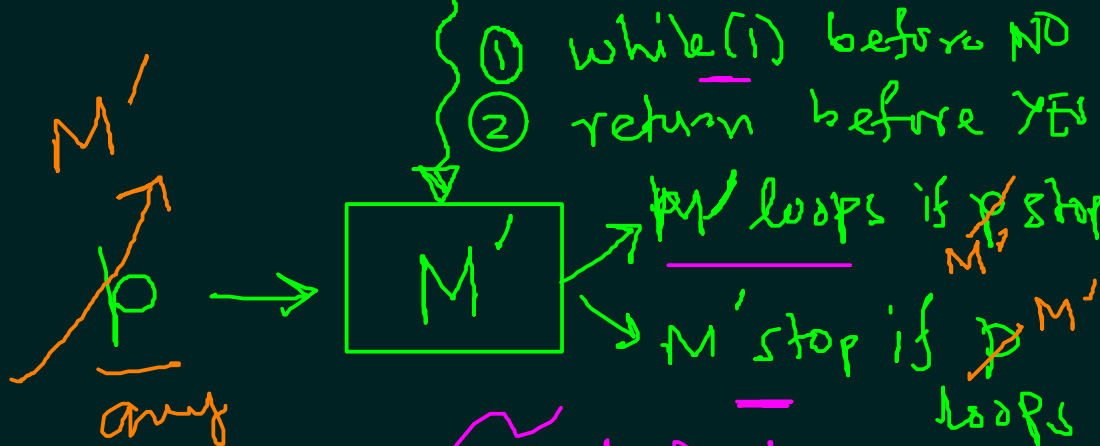
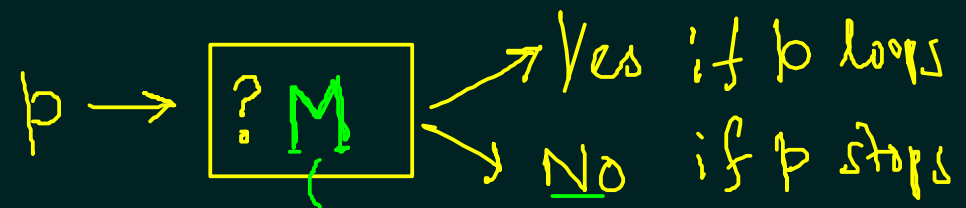
Gregor Cantor

$\hookrightarrow$  Solve countably infinite / finite

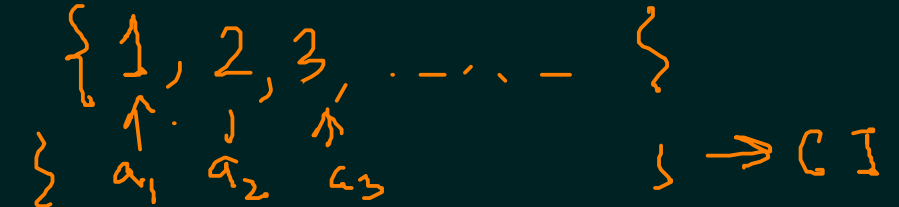
SP	$I_1$	$I_2$	$I_3$	$I_4$	...
$P_1$	0	1	0	1	
$P_2$	1	1	1	0	
$P_3$	0	0	1	1	
$\vdots$	1	0	1	1	
$OP_1$	1	0	0	0	...

Diagonalization

# Solve



Contradiction



$\Rightarrow$  # Integers

$\hookrightarrow$  permutation of  $I$

$= P(I) = |\text{Reals}|$

$$\frac{\# \text{Solve}}{\# \text{Unsolve}} = \frac{\# \text{Integers}}{\# \text{Reals}}$$



decidables  
 (solvable)

Syllabus  
 course

Model  
 (Machines)  
 FA/RL  
 CFG/CFL  
 TM

Complexity  
 (Time  
 + Space)

Computability  
 Decidability  
 Undecidability

F O  
 G S  
 Logic

$\Sigma^*$   
 Language  
 Representation

Deduction  
 Reasoning

D&G  
 Dy Gr

Algorithm  
 (procedure)

(Discrete  
 Structure)

X