TUTORIAL – 5

(TIME & SPACE COMPLEXITY)

Problem-1

For each of the following statements, answer True, False or Open-Question according to our current state of knowledge of complexity theory, as described in class. Give brief justifications for your answers.

(a) $P \subseteq TIME(n^{2026})$?

(b) $SAT \leq_P \overline{SAT}$?

(c) $HAMPATH \leq_P PATH$?

(d) $PATH \leq_P \overline{PATH}$?

(e) $NSPACE(n^{2026}) \subseteq PSPACE$?

Problem-2

Prove that the following languages (defined over graphs) are in P.

(a) BIPARTITE : the set of all bipartite graphs, i.e., G = (V, E) ∈ BIPARTITE if V can be partitioned into two sets V1, V2 such that every edge in E is adjacent to a vertex in V1 and a vertex in V2 (no edge falls inside V1 or V2).

(b) TRIANGLE-FREE : the set of all graphs that do not contain a triangle (where triangle is a set of three distinct vertices that are mutually connected).

Problem-3

Let DOUBLE-SAT = $\{\langle \varphi \rangle \mid \varphi$ is a CNF formula having at least two satisfying assignments$\}$. Show that DOUBLE-SAT is NP-complete.

Problem-4

A vertex cover in a graph $G = (V, E)$ is a set of vertices $S \subseteq V$ such that every edge of G is incident on at least one vertex in S. Show that the following language is NP-complete.

VERTEX-COVER = $\{(G, k) \mid$ graph G has a vertex cover of size $\leq k\}$

Problem-5

Let S be a set and let C = { X_1, X_2, ..., X_n } be a collection of n subsets of S (for each i ∈ [1,n], $X_i \subseteq S$). A set S', with S' ⊆ S, is called a hitting set for C if every subset in C contains at least one element in S', i.e., $|X\ i \cap S'| \geq 1$ for each i ∈ [1,n]. Let HITSET = {⟨C, k⟩ | C has a hitting set of size k}. Prove that HITSET is NP-complete.

Example: S = {a,b,c,d,e,f,g}, C = {{a,b,c},{d,a},{d,e,f},{g}}

- k = 2, no hitting sets exist.
- k = 3, S' = {a, d, g} (other choices exist).

Problem-6

Prove the following:

(a) Prove that P = coP and P $\subseteq$ NP $\cap$ coNP.

(b) Assuming NP $\neq$ coNP, show that no NP-complete problem can be in coNP.

(c) Show that the halting problem is NP-hard.

(d) Prove that, If every NP-hard language is PSPACE-hard, then PSPACE = NP.

Problem-7

In the generalized version of the game Tic-Tac-Toe, 2 players places marks X (crosses) and O (noughts) on an m x n grid. A player wins if she is the first to place k marks in a row, column or diagonal. The game ends in a draw if no such sequence is present when all the mn cells of the grid are filled. Assuming that X always starts, show that the following language, GTICTACTOE, is in PSPACE.

GTICTACTOE = { ⟨m, n, k, c⟩ | c is an intermediate configuration on the m x n board with next move by X and ∃ a winning strategy for X }

THANK YOU !