

# **TUTORIAL - 3B**

**(TURING MACHINES AND  
UNRESTRICTED GRAMMARS)**



# Problem-1

Design Turing Machines (TMs) as well as unrestricted grammars for the following languages:

$$L_1 = \{ w \in \{a,b,c\}^* \mid \#a(w) = \#b(w) = \#c(w) \}$$

$$L_2 = \{ ww \mid w \in \{a,b\}^* \}$$

$$L_3 = \{ a^i b^j c^k d^l \mid i=k \text{ and } j=l \}$$

$$L_a = \{ a^n w c^n \mid w \in \{a,b,c\}^*, n \geq 0, \text{ and } \#a(w) = n \}$$

$$L_b = \{ a^n w c^n \mid w \in \{a,b,c\}^*, n \geq 0, \text{ and } \#b(w) = n \}$$

$$L_c = \{ a^n w c^n \mid w \in \{a,b,c\}^*, n \geq 0, \text{ and } \#c(w) = n \}$$

## Problem-2

Consider the unrestricted grammar over the singleton alphabet  $\Sigma = \{a\}$ , having the start symbol  $S$ , and with the following productions.

$$S \rightarrow AS \mid aT, \quad Aa \rightarrow aaaA, \quad AT \rightarrow T, \quad T \rightarrow \varepsilon$$

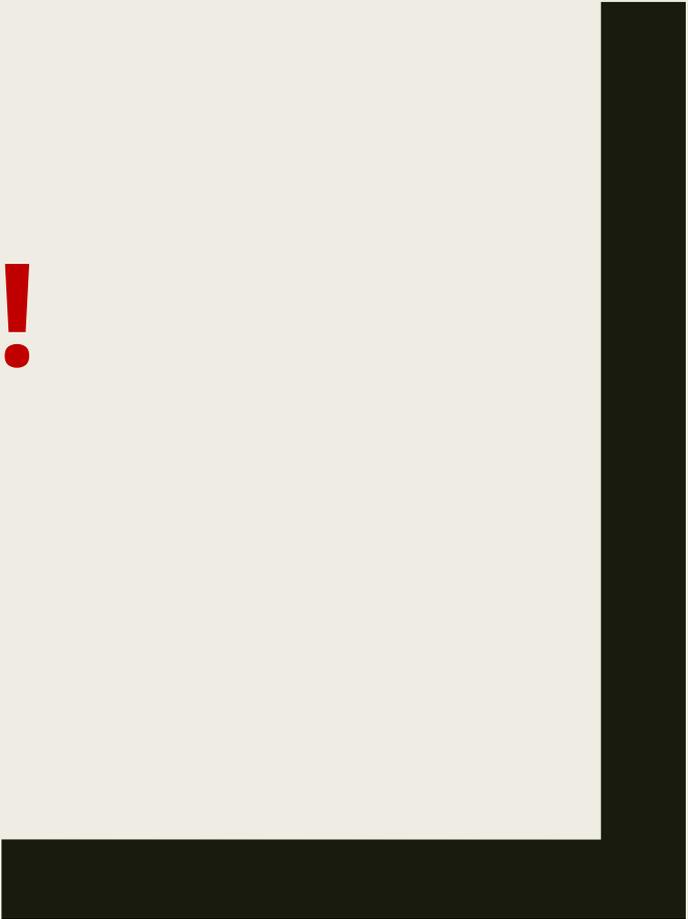
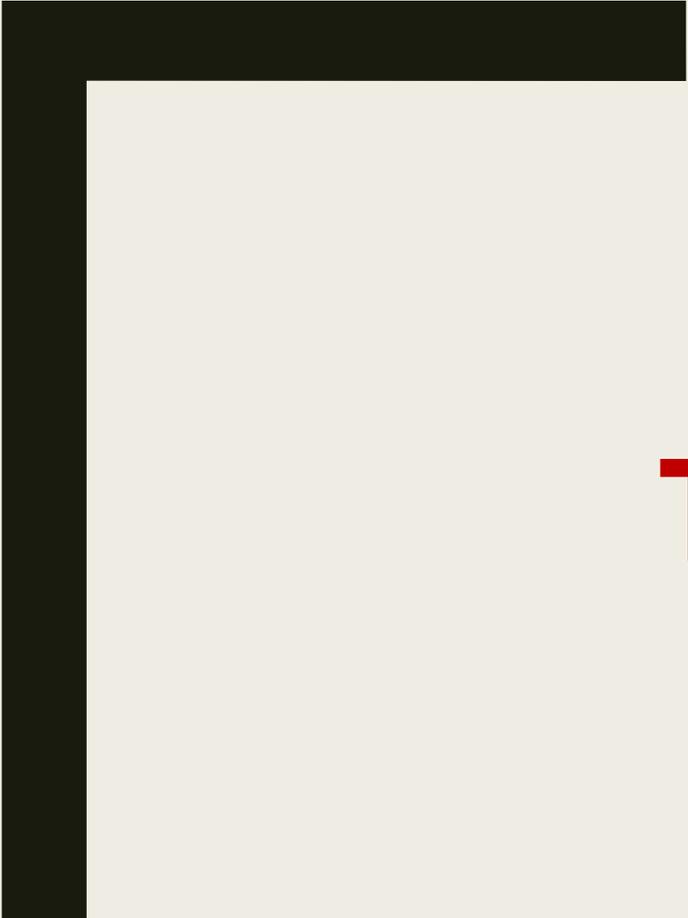
What is the language generated by this unrestricted grammar? Justify.

## Problem-3

**Prove that, a language  $L$  is recursive if and only if there is an enumeration machine enumerating the strings of  $L$  in a non-decreasing order of length (string of the same length may be arranged in lexicographic order). For example, strings of  $\{0, 1\}^*$  would be arranged as 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 100, 101, 110, 111, . . . .**

## Problem-4

**Prove that any grammar, defined over non-terminals  $N$  and terminals  $\Sigma$ , can be converted to an equivalent grammar with rules of the form,  $aA\gamma \rightarrow a\beta\gamma$  for  $A \in N$  and  $a, \beta, \gamma \in (\Sigma \cup N)^*$ .**



**THANK YOU !**