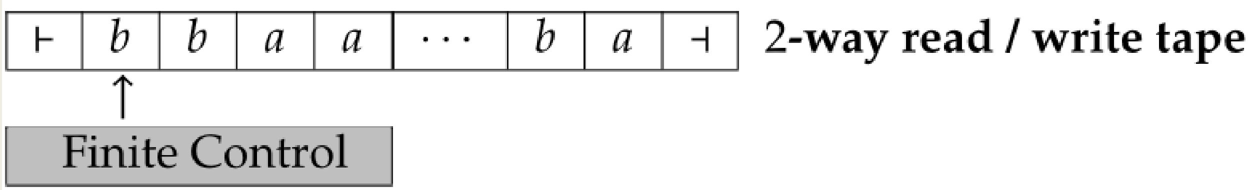
TUTORIAL – 3A

(TURING MACHINES AND EQUIVALENT MODELS)

Problem-1

A linear bounded automaton (LBA) is exactly like a 1-tape TM, except that the input string $x \in \Sigma^*$ is enclosed in left and right endmarkers $\vdash$ and $\dashv$ which may not be overwritten. The machine is constrained never to move left of $\vdash$ or right of $\dashv$. It is allowed to read/write between these markers.



(a) Give a rigorous formal definition of deterministic linearly bounded automata, including a definition of configurations and acceptance.

(b) Let M be an LBA with state set Q of size k and tape alphabet Γ of size m. How many possible configurations are there on input x with $|x| = n$?

(c) Argue that it is possible to detect in finite time whether an LBA loops on a given input. (Hint: Think of using Part (b).)

Problem-2

Design an NTM to accept the language,

$\{wxyxz \mid w, x, y, z \in \{0, 1\}^*, |x| = 2026\}$.

Problem-3

A TM $\mathscr{M}$ has a two-way infinite tape. Initially, all cells on the tape are blank. Only one cell is storing the symbol #. The head of $\mathscr{M}$ is pointing to a blank. The task of $\mathscr{M}$ is to locate the cell storing #.

Propose a strategy for doing this,

(a) if $\mathscr{M}$ is a DTM,

(b) if $\mathscr{M}$ is a NTM.

Problem-4

A Jump Turing Machine (JTM) $J = (Q, \Sigma, \Gamma, \delta, \vdash, \sqcup, s, t, r)$ is like a standard one-tape Turing Machine (TM) with the only exception that each transition of J is of the form $\delta(p,A) = (q,B,m)$, where $p,q \in Q$, and $A,B \in \Gamma$, and $m \in Z$.

This means that if the finite control of J is in the state p and the head of J scans the tape symbol A, then the state changes to q, the content of the tape cell is changed from A to B, and the head jumps by m (integer) cells relative to the current position.

If $m = 0$, the head stays at the current cell. If $m > 0$, the head makes a right jump. If $m < 0$, the head makes a left jump with the understanding that if the head is at position i on the tape and $|m| > i$, then the head goes to the leftmost cell (which stores the left end-marker $\vdash$). Also assume that if A is $\vdash$, then $m \geq 0$.

Prove that a JTM is equivalent to a TM.

Problem-5

Show that the class of recursively enumerable sets is closed under union and intersection.

THANK YOU !