INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Stamp / Signature of the Invigilator

EXAMINATION (Mid Semester)	**SEMESTER (Spring 2025-2026)**

Roll Number										**Section**		**Name**	

Subject Number	C	S	2	1	2	0	4	**Subject Name**	*FORMAL LANGUAGE AND AUTOMATA THEORY*

Department / Center of the Student		**Additional sheets**	

Important Instructions and Guidelines for Students

1. You must occupy your seat as per the Examination Schedule/Sitting Plan.
2. Do not keep mobile phones or any similar electronic gadgets with you even in the switched off mode.
3. Loose papers, class notes, books or any such materials must not be in your possession, even if they are irrelevant to the subject you are taking examination.
4. Data book, codes, graph papers, relevant standard tables/charts or any other materials are allowed only when instructed by the paper-setter.
5. Use of instrument box, pencil box and non-programmable calculator is allowed during the examination. However, exchange of these items or any other papers (including question papers) is not permitted.
6. Write on both sides of the answer script and do not tear off any page. **Use last page(s) of the answer script for rough work.** Report to the invigilator if the answer script has torn or distorted page(s).
7. It is your responsibility to ensure that you have signed the Attendance Sheet. Keep your Admit Card/Identity Card on the desk for checking by the invigilator.
8. You may leave the examination hall for wash room or for drinking water for a very short period. Record your absence from the Examination Hall in the register provided. Smoking and the consumption of any kind of beverages are strictly prohibited inside the Examination Hall.
9. Do not leave the Examination Hall without submitting your answer script to the invigilator. **In any case, you are not allowed to take away the answer script with you.** After the completion of the examination, do not leave the seat until the invigilators collect all the answer scripts.
10. During the examination, either inside or outside the Examination Hall, gathering information from any kind of sources or exchanging information with others or any such attempt will be treated as **'unfair means'**. Do not adopt unfair means and do not indulge in unseemly behavior.

Violation of any of the above instructions may lead to severe punishment.

Signature of the Student

To be filled in by the examiner

Question Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**
Marks Obtained											

Marks obtained (in words)	**Signature of the Examiner**	**Signature of the Scrutineer**

Indian Institute of Technology Kharagpur
Department of Computer Science and Engineering

Formal Language and Automata Theory (CS21204) **Spring 2025-2026**

21-February-2026 (FN) **Mid-Semester Examination** Maximum Marks: 60

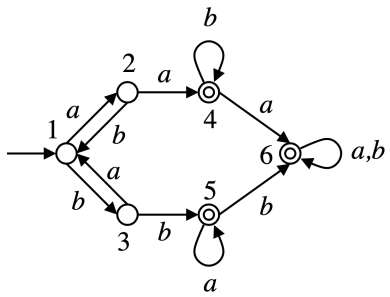
Instructions:

- Write your answers in the question paper itself. Be brief and precise. Answer *all* questions.
- Write the answers only in the respective spaces provided. The last three blank pages may be used for rough work or leftover answers.
- In case you may need more space/pages, please ask for additional sheets in the exam hall and attach the same with this booklet while submitting.
- If you use any algorithm / result / formula covered in the class, just mention it, do not elaborate (unless the same thing has been explicitly asked to answer in the question).

— Question Paper Starts from Next Page —

Q1. [Finite Automata] **15 marks**

(a)



Consider the deterministic finite automaton (DFA) in the left where the six states are numbered as $\{1,2,3,4,5,6\}$ and transitions are given over the alphabet $\Sigma = \{a,b\}$. Use the DFA state-minimization algorithm to convert this DFA to an equivalent DFA with the minimum possible number of states. Show all the steps in details. Also draw the minimum-state DFA or the quotient automaton. **(6)**

Solution:

(b) Let L be the language, $L = \{w \in \{a,b\}^* \mid w$ contains an equal number of occurrences of ab and $ba\}$. For example, $ababa \in L$ (two occurrences of ab, and two of ba), whereas $bbaba \notin L$ (one occurrence of ab, and two of ba). Answer the following questions in parts (i) and (ii).

(i) Give a regular expression for the language L with proper justifications. **(4)**

Solution:

(ii) Design a minimum-state deterministic finite automaton (DFA) to accept L. Show only the state-transition diagram, not its mathematical definition. **(5)**

Solution:

Q2. [Regular Languages] **15 marks**

For the following problems in parts (a), (b) and (c) under this question, we define the languages L over alphabet $\Sigma = \{0,1\}$. Prove or disprove whether $L \in \mathsf{RL}$. *No credits will be given without justifications.*

Note: RL denotes the class of regular languages. For proving $L \in \mathsf{RL}$, you must present a Regular-Expression / DFA / NFA / ε-NFA (any <u>one</u> of these) that accepts L. However, to disprove (i.e., proving $L \notin \mathsf{RL}$), you <u>must</u> use pumping lemma or closure properties for regular languages to contradict.

(a) $L = \left\{0^k w 1^k \;\middle|\; k \geq 1, w \in \{0,1\}^*\right\}$. **(5)**

Solution:

(b) $L = \left\{0^k 1 w 1^k \;\middle|\; k \geq 1, w \in \{0,1\}^*\right\}$. **(5)**

Solution:

(c) $L = \{ww \mid w \in \{0,1\}^* \text{ and } w \text{ contains at least one 0 and at least one 1}\}$. **(5)**

Solution:

Q3. [Context-free Grammars and Derivations] **8 marks**

Consider the following context-free grammar (CFG) $G = (V, \Sigma, P, S)$ with $V = \{S\}, \Sigma = \{a, +, \times\}$ to generate arithmetic expressions in one variable a involving addition and multiplication operations only. Here, S is the start symbol and P is the set of following productions/rules:

$$S \rightarrow a \mid S+S \mid S \times S$$

(a) Draw all possible derivation (parse) trees for the string $a+a \times a+a$ following this grammar. **(5)**

Solution:

(b) Design an *unambiguous* CFG to generate the same language. Force the operations to be evaluated from left to right. This means that $+$ and $\times$ are given the same precedence and left-to-right associativity. For example, $a+a \times a+a$ is to be interpreted as $((a+a) \times a)+a$. **(3)**

Solution:

Q4. [Context-free Languages] **14 marks**

For the following problems in parts (a) and (b) under this question, we define languages L over alphabet $\Sigma = \{a, b, c\}$. Prove or disprove whether $L \in \mathsf{CFL}$. *No credits will be given without justifications.*

Note: CFL denotes the class of context-free languages. For proving $L \in \mathsf{CFL}$, you must present a context-free grammar (CFG) and a push-down automaton (PDA) (both are asked here) accepting L. To disprove (i.e., to prove $L \notin \mathsf{CFL}$), you must use pumping lemma for context-free languages.

(a) $L = \left\{ a^i b^j c^j \;\middle|\; i, j \geq 0 \text{ and } i \geq j \right\}$. **(7)**

Solution:

(b) $L = \left\{ a^i (bc)^j \;\middle|\; i, j \geq 0 \text{ and } i \geq j \right\}$. **(7)**

Solution:

Q5. [Regular Grammars] **8 marks**

Consider (again) the language, $L = \{w \in \{a,b\}^* \mid w$ contains an equal number of occurrences of ab and $ba\}$, introduced in **Q1(b)**. Answer the following questions. *You may use your DFA presented in* **Q1b(ii)**.

(a) Construct a *right-linear grammar* which can generate L. **(4)**

Solution:

(b) Construct a *left-linear grammar* which can generate L. **(4)**

Solution:

— Question Paper Ends Here —