



INDIAN INSTITUTE OF TECHNOLOGY  
KHARAGPUR

Stamp / Signature of the Invigilator

EXAMINATION ( End Semester )

SEMESTER ( Spring 2025-2026 )

Roll Number

Section

Name

Subject Number

C S 2 1 2 0 4

Subject Name

FORMAL LANGUAGE AND AUTOMATA THEORY

Department / Center of the Student

Additional sheets

**Important Instructions and Guidelines for Students**

1. You must occupy your seat as per the Examination Schedule/Sitting Plan.
2. Do not keep mobile phones or any similar electronic gadgets with you even in the switched off mode.
3. Loose papers, class notes, books or any such materials must not be in your possession, even if they are irrelevant to the subject you are taking examination.
4. Data book, codes, graph papers, relevant standard tables/charts or any other materials are allowed only when instructed by the paper-setter.
5. Use of instrument box, pencil box and non-programmable calculator is allowed during the examination. However, exchange of these items or any other papers (including question papers) is not permitted.
6. Write on both sides of the answer script and do not tear off any page. **Use last page(s) of the answer script for rough work.** Report to the invigilator if the answer script has torn or distorted page(s).
7. It is your responsibility to ensure that you have signed the Attendance Sheet. Keep your Admit Card/Identity Card on the desk for checking by the invigilator.
8. You may leave the examination hall for wash room or for drinking water for a very short period. Record your absence from the Examination Hall in the register provided. Smoking and the consumption of any kind of beverages are strictly prohibited inside the Examination Hall.
9. Do not leave the Examination Hall without submitting your answer script to the invigilator. **In any case, you are not allowed to take away the answer script with you.** After the completion of the examination, do not leave the seat until the invigilators collect all the answer scripts.
10. During the examination, either inside or outside the Examination Hall, gathering information from any kind of sources or exchanging information with others or any such attempt will be treated as '**unfair means**'. Do not adopt unfair means and do not indulge in unseemly behavior.

**Violation of any of the above instructions may lead to severe punishment.**

Signature of the Student

*To be filled in by the examiner*

Question Number

1

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Total

Marks Obtained

Marks obtained (in words)

Signature of the Examiner

Signature of the Scrutineer

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**Indian Institute of Technology Kharagpur**  
**Department of Computer Science and Engineering**

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**Formal Language and Automata Theory (CS21204)**

**Spring 2025-2026**

21-April-2026

**End-Semester Examination**

Maximum Marks: 60

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**Instructions:**

- Write your answers in the question paper itself. Be brief and precise. Answer *all* questions.
  - Write the answers only in the respective spaces provided. The last three blank pages may be used for rough work or leftover answers.
  - In case you may need more space/pages, please ask for additional sheets in the exam hall and attach the same with this booklet while submitting.
  - If you use any algorithm / result / formula covered in the class, just mention it, do not elaborate (unless the same thing has been explicitly asked to answer in the question).
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**Q1. [ Turing Machines and Unrestricted Grammars ]**

**15 marks**

(a) Write an unrestricted grammar to generate the following language:

$$L_1 = \{a^i b^j c^k d^l \mid i = k \text{ and } j = l\}, \quad \text{where } i, j, k, l \text{ are non-negative integers.}$$

Mention the start symbol of your grammar. Use upper-case letters for non-terminal symbols. (5)

**Solution:**

- (b) Show a derivation of the string  $a^2b^3c^2d^3$  according to your grammar. Clearly mention the exact production that you have used in each step of your derivation. (5)

**Solution:**

(c) Present an informal algorithmic description of a Turing Machine that *decides*  $L_1$ .

(5)

**Solution:**

**Q2. [ Decidable Languages ]****14 marks**

Prove that the following languages are decidable. Provide only high-level descriptions of deciders.

- (a)  $\text{EMPTY}_{\text{DFA}} = \{ \langle A \rangle \mid A \text{ is a DFA and } \mathcal{L}(A) = \emptyset \}$ . (3)

**Solution:**

- (b)  $\text{SUBSET}_{\text{DFA}} = \{ \langle D1, D2 \rangle \mid D1, D2 \text{ are DFA with } \mathcal{L}(D1) \subseteq \mathcal{L}(D2) \}$ . (3)

**Solution:**

(c)  $\text{FINITE}_{\text{PDA}} = \{ \langle P \rangle \mid P \text{ is a PDA with } \mathcal{L}(P) \text{ being finite} \}$ . (5)

Hint: Use the fact that  $A_{\text{PDA}} = \{ \langle P, w \rangle \mid P \text{ is a PDA that accepts string } w \}$  is decidable.

**Solution:**

(d)  $\text{MOVE}_{\text{TM}} = \{ \langle M, n \rangle \mid M \text{ is a TM that makes at least } n \text{ moves on some input} \}$ . (3)

**Solution:**

**Q3. [ Semi-decidable Languages and Problem Reduction ]**

**11 marks**

Consider the following language,  $L_2 = \{ \langle M \rangle \mid M \text{ is a TM that halts on the input } 010101 \}$ .

Prove the following assertions:

- (a) Prove that  $L_2$  is not recursive (i.e., not Turing-decidable). (5)

Hint: Reduce from membership problem,  $MP = \{ \langle N, w \rangle \mid N \text{ is a TM which accepts the input string } w \}$ .

**Solution:**

(b) Prove that  $L_2$  is recursively enumerable (i.e., Turing-recognizable).

(3)

**Solution:**

(c) Consider the complement of  $L_2$ :

$$\overline{L_2} = \{ \langle M' \rangle \mid M' \text{ is a TM which does not halt on the input } 010101 \}.$$

Prove that  $\overline{L_2}$  is not recursively enumerable (i.e., not Turing-recognizable).

(3)

**Solution:**

**Q4. [ Undecidable Languages and Problem Reduction ]**

**10 marks**

Answer the following questions using the problem reduction paradigm.

(a) Consider the following language:

$$L_3 = \{ \langle M \rangle \mid M \text{ is a TM which halts on every input} \}.$$

Show a reduction from  $\overline{L_2}$  to  $L_3$  to prove that  $L_3$  is not recursively enumerable (i.e., not Turing-recognizable). Note that the language  $\overline{L_2}$  is introduced already in **Q3(c)**. (5)

**Solution:**

(b) Consider the complement of  $L_3$ :

$$\overline{L_3} = \{ \langle \overline{M} \rangle \mid \overline{M} \text{ is a TM which does not halt on every input} \}.$$

Show a reduction from  $\overline{L_2}$  to  $\overline{L_3}$  to prove that  $\overline{L_3}$  is also not recursively enumerable (i.e., not Turing-recognizable). Note that the language  $\overline{L_2}$  is introduced already in **Q3(c)**. **(5)**

**Solution:**

**Q5. [ Rice's Theorem and Properties of R.E. Languages ]**

**10 marks**

Which of the following are properties of recursively enumerable sets? First, supply an answer Yes/No. If the answer is *Yes*, mention whether the property is trivial and/or monotone. If the answer is *No*, supply a one-line justification. In each case, the property is specified by a Turing machine  $M$ . (5 × 2)

(a)  $M$  accepts  $\varepsilon$ .

**Solution:**

(b)  $M$  (explicitly) rejects  $\varepsilon$ .

**Solution:**

(c)  $M$  halts on  $\varepsilon$ .

**Solution:**

(d)  $\mathcal{L}(M)$  is a context-free language.

**Solution:**

(e)  $\mathcal{L}(M)$  contains a context-free language.

**Solution:**

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— Question Paper Ends Here —

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