CS21204 : Formal Language and Automata Theory (Spring 2026)

Class Test 2

26-Mar-2026 (Thursday) | Maximum Marks: 30 | 06:30pm – 07:45pm

---

Roll: ____________________ **Name:** ____

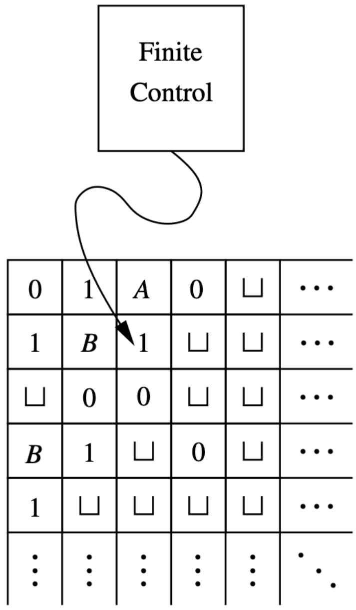
[*Write your answers in question paper. Answer* *all* *questions. Be brief and precise.*]

Q1. [Turing Machines]

Consider the language $L = \left\{ a^{n^2} \mid n \geq 1 \right\}$ defined over the alphabet $\Sigma = \{a\}$. Describe a (total) Turing machine that decides the language L. **(7)**

Solution:

Q2. [Equivalent TM Models and R.E. Languages]



Consider a Turing machine M with a two-dimensional tape as shown in the adjacent figure. The initial input string is provided horizontally at the topmost row starting from the first column, the finite control of M is in the start state, and the head is positioned at the top-left cell of the tape. Each move of M is dependent on the state of the finite control and on the tape symbol scanned by the head. During each move the finite control switches to a new state (which may be the same as the old state), the tape symbol currently scanned by the head is overwritten by a new symbol (which may be the same as the old symbol), and the head moves by one cell in any one of the four directions north, east, west and south. M accepts by entering a final state.

Show that $\mathscr{L}(M)$ is R.E. (i.e., Turing-recognizable). **(10)**

Solution:

Q2 Solution (continued):

Q3. [Properties of Recursive Languages]

(a) Prove that Turing-recognizable (R.E.) languages are closed under union. **(4)**

Solution:

(b) Let $L_1,\ L_2,\ \ldots,\ L_n$ be pairwise disjoint Turing-recognizable (R.E.) languages over the same alphabet Σ. Suppose that $\bigcup_{i=1}^{n} L_i = \Sigma^*$. Prove that each L_i is Turing-decidable (Recursive). **(4)**

Solution:

Q4. [Push-Down Automaton]

Let $\alpha = a_1 a_2 \ldots a_n$ be a string of length n. A string β is called a prefix of α if $\beta = a_1 a_2 \ldots a_i$ for some $i \in \{0, 1, 2, \ldots, n\}$ (the case $i = 0$ corresponds to $\beta = \varepsilon$). Consider the following language,

$$L_0 = \left\{ \alpha \in \{0,1\}^* \;\middle|\; \text{no prefix of } \alpha \text{ contains less 0's than 1's} \right\}$$

Prove that L_0 is context-free by formally presenting a push-down automaton (PDA) that accepts L_0. **(5)**

Solution:

— End of Question Paper —