

Roll: _____ Name: _____

[Write your answers in question paper. Answer all questions. Be brief and precise.]

Q1. [Languages – Strings and Operations]

Let us define languages L_1, L_2, L_3 as sets of strings over the singleton alphabet $\Sigma = \{c\}$. Present examples (with proper justifications) satisfying the conditions as specified below.

(a) Give an example of a language $L_1 \subseteq \Sigma^*$ such that $L_1^* = L_1^+$. (2)

Solution:

(b) Give an example of a language $L_2 \subseteq \Sigma^*$ such that $L_2 = L_2^*$. (2)

Solution:

(c) Give an example of a language $L_3 \subseteq \Sigma^*$ such that L_3^* is finite. (2)

Solution:

Q2. [NFA and DFA – Construction and Properties]

Consider the following language over $\{a, b\}$ and answer the questions asked below.

$$L_y = \{ y \mid y \in \{a, b\}^* \text{ and } y \text{ does not start and end with the same symbol} \}$$

(a) Give a regular expression for the language L_y with a brief justification. (2)

Solution:

(b) From the regular expression formed in part (a), *algorithmically* construct a non-deterministic finite automaton (NFA) with ϵ -transitions accepting the language L_y and show stepwise how you proceed to do the same. Show state-transition diagram(s), not its mathematical definition(s). (4)

Solution:

(c) Present a 4-state NFA N (without any ϵ transition) accepting L_y . Show the state-transition diagram for N , not its mathematical definition. (2)

Solution:

(d) Apply subset construction procedure to convert the above NFA N , that you produced in part (c), to an equivalent DFA D . Show state-transition diagram for D , not its mathematical definition. (3)

Solution:

(e) Modify your DFA D , that you constructed in part (d), to create \bar{D} so that it accepts the complement of L_y , i.e. $\bar{L}_y = \{a,b\}^* - L_y$. Show the state-transition diagram for \bar{D} , not its mathematical definition. (2)

Solution:

Q3. [Regular Expression and DFA]

Consider the following language over $\{0,1,2\}$ and answer the questions asked below.

$$L_z = \{ z \mid z \in \{0,1,2\}^* \text{ and } z \text{ contains an even number of 0s, or } z \text{ contains exactly two 1s} \}$$

(a) Give a regular expression for the language L_z with a brief justification. (2)

Solution:

(b) Present a DFA accepting the language L_z . Show the state-transition diagram for the DFA, not its mathematical definition. (4)

Solution:

Q4. [Regular Languages]

Suppose that $L \subseteq \{0, 1\}^*$ is a regular language. Let us define the following language (set),

$$\text{DEL}_1(L) = \{uv \mid u1v \in L \text{ and } u, v \in \{0, 1\}^*\}$$

The set $\text{DEL}_1(L)$ essentially consists of all strings that can be obtained from strings in L by deleting *exactly* one 1. Prove that $\text{DEL}_1(L)$ is also regular. (Hint: Use non-determinism) (5)

Solution:

