INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR	
	Stamp / Signature of the Invigilator

EXAMINATION (Mid Semester)	**SEMESTER (Spring 2025-2026)**

Roll Number									**Section**		**Name**	
Subject Number	C	S	6	0	0	2	0	**Subject Name**	***FOUNDATIONS OF ALGORITHM DESIGN AND MACHINE LEARNING***			
Department / Center of the Student										**Additional sheets**		

Important Instructions and Guidelines for Students

1. You must occupy your seat as per the Examination Schedule/Sitting Plan.
2. Do not keep mobile phones or any similar electronic gadgets with you even in the switched off mode.
3. Loose papers, class notes, books or any such materials must not be in your possession, even if they are irrelevant to the subject you are taking examination.
4. Data book, codes, graph papers, relevant standard tables/charts or any other materials are allowed only when instructed by the paper-setter.
5. Use of instrument box, pencil box and non-programmable calculator is allowed during the examination. However, exchange of these items or any other papers (including question papers) is not permitted.
6. Write on both sides of the answer script and do not tear off any page. **Use last page(s) of the answer script for rough work.** Report to the invigilator if the answer script has torn or distorted page(s).
7. It is your responsibility to ensure that you have signed the Attendance Sheet. Keep your Admit Card/Identity Card on the desk for checking by the invigilator.
8. You may leave the examination hall for wash room or for drinking water for a very short period. Record your absence from the Examination Hall in the register provided. Smoking and the consumption of any kind of beverages are strictly prohibited inside the Examination Hall.
9. Do not leave the Examination Hall without submitting your answer script to the invigilator. **In any case, you are not allowed to take away the answer script with you.** After the completion of the examination, do not leave the seat until the invigilators collect all the answer scripts.
10. During the examination, either inside or outside the Examination Hall, gathering information from any kind of sources or exchanging information with others or any such attempt will be treated as **'unfair means'**. Do not adopt unfair means and do not indulge in unseemly behavior.

Violation of any of the above instructions may lead to severe punishment.

Signature of the Student

To be filled in by the examiner

Question Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**
Marks Obtained											

Marks obtained (in words)	**Signature of the Examiner**	**Signature of the Scrutineer**

Indian Institute of Technology Kharagpur
Department of Computer Science and Engineering

Foundations of Algorithm Design and Machine Learning (CS60020) **Spring 2025-2026**

22-February-2026 (AN) **Mid-Semester Examination** Maximum Marks: 60

Instructions:

- Write your answers in the question paper itself. Be brief and precise. Answer *all* questions.
- Write the answers only in the respective spaces provided. The last three blank pages may be used for rough work or leftover answers.
- In case you may need more space/pages, please ask for additional sheets in the exam hall and attach the same with this booklet while submitting.
- If you use any algorithm / result / formula covered in the class, just mention it, do not elaborate (unless the same thing has been explicitly asked to answer in the question).

— Question Paper Starts from Next Page —

Q1. **[Recursive Formulation and Algorithm Complexity]** **12 marks**

Consider an $n \times n$ 2-dimensional array called A, consisting of only 0s and 1s, where every row (from left to right) and every column (from top to bottom) have a sequence of 0s followed by a sequence of 1s. Some rows or columns may have only 0s and some rows or columns may have only 1s. However, in no case will we have a row or a column where a 0 follows a 1. You are to find the total number of 1s in the array A using an efficient algorithm. For this, answer the following questions.

(a) Present a recursive algorithm (using pseudo-code) to solve the problem efficiently. Clearly define the arguments of the recursive function, and explain them. Highlight the base and recursive portions of your algorithm with proper comments. Clearly indicate the return value. **(5)**

Solution:

To be given.

(b) Show the working of your algorithm on the following example of a 5×5 array of elements. **(3)**

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Solution:

To be given.

(c) Present (with a brief justification) the asymptotic time and space complexity of your algorithm. **(2+2)**

Solution:

To be given.

Q2. **[Dynamic Programming]** **18 marks**

Consider the problem of finding the Longest Common Subsequence (LCS) from three lists, namely L_1, L_2 and L_3, where each list represents a sequence of characters. For this, answer the following questions.

(a) Present an example to show that solving the LCS of 3 lists cannot be guaranteed by finding the LCS of any 2 lists first and then using that result with the third list to get the final solution. **(4)**

Solution:

To be given.

(b) Present the recursive formulation to solve the LCS of 3 lists. Explain the steps with comments. **(4)**

Solution:

To be given.

(c) Convert the solution to a top-down dynamic programming algorithm (using *memoization*) and explain the algorithm with comments. Analyze its time and space complexity. **(4+2)**

Solution:

To be given.

(d) Show the working of your proposed algorithm on an example of your choice with each list having at least 5 characters. **(4)**

Solution:

To be given.

Q3. **[Naïve Bayes Classifier]** **10 marks**

A dataset of 6 students containing 4 attributes and 1 outcome, Placed (P), is shown on right. Note that, $\mathbb{P}(P = Yes) = \mathbb{P}(P = No) = \frac{1}{2}$. Here, Course-Type ($C$) and Attend-Class ($A$) are discrete attributes, whereas Study-Hours (S) and Marks (M) are continuous attributes. Assume that all continuous attributes follow normal (gaussian) distribution having probability density function,

Course Type	Attend Class	Study Hours	Marks (%)	Placed (Y/N)
Theory	*Average*	5	74	*No*
Theory	*Low*	10	70	*No*
Theory	*High*	8	92	*Yes*
Lab	*Average*	9	83	*Yes*
Lab	*High*	12	66	*No*
Lab	*Low*	16	87	*Yes*

$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, where μ and σ are the mean and standard deviation. Accordingly, the mean and variance of a set of n i.i.d. samples, $x_1, x_2, \ldots, x_n \sim p(x)$ can be computed (respectively) as follows:

$$[\texttt{mean}] \quad \mu = \frac{1}{n}\sum_{i-1}^{n} x_i \qquad and \qquad [\texttt{variance}] \quad \sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

Your task is to predict whether two new students, having the following attribute values, will be placed.

X_1 : $\langle$ Course-Type $= Theory$, Attend-Class $= Average$, Study-Hours $= 6$, Marks $= 75$ $\rangle$

X_2 : $\langle$ Course-Type $= Lab$, Attend-Class $= Low$, Study-Hours $= 9$, Marks $= 82$ $\rangle$

Applying (Gaussian) Naïve Bayes method, answer every step of computation as asked below.

(a) Compute the required conditional probabilities of the discrete attributes given the outcome. **(2)**

Solution:

[0.5 mark]

$$\mathbb{P}(C = Theory \mid P = Yes) = \frac{1}{3} \qquad \mathbb{P}(A = Low \mid P = Yes) = \frac{1}{3}$$

[0.5 mark]

$$\mathbb{P}(C = Theory \mid P = No) = \frac{2}{3} \qquad \mathbb{P}(A = Low \mid P = No) = \frac{1}{3}$$

[0.5 mark]

$$\mathbb{P}(C = Lab \mid P = Yes) = \frac{2}{3} \qquad \mathbb{P}(A = Average \mid P = Yes) = \frac{1}{3}$$

[0.5 mark]

$$\mathbb{P}(C = Lab \mid P = No) = \frac{1}{3} \qquad \mathbb{P}(A = Average \mid P = No) = \frac{1}{3}$$

(b) Compute the required mean and variance for the continuous attributes given the outcome. **(2)**

Solution:

[0.5 mark]

$$\mu_{S|P=Yes} = \frac{8+9+16}{3} \approx 11$$

$$\mu_{S|P=No} = \frac{5+10+12}{3} \approx 9$$

[0.5 mark]

$$\sigma_{S|P=Yes} = \sqrt{\frac{(-3)^2+(-2)^2+(5)^2}{3}} \approx \sqrt{12.67} \approx 3.559$$

$$\sigma_{S|P=No} = \sqrt{\frac{(-4)^2+(1)^2+(3)^2}{3}} \approx \sqrt{8.67} \approx 2.944$$

[0.5 mark]

$$\mu_{M|P=Yes} = \frac{92+83+87}{3} \approx 87.33$$

$$\mu_{M|P=No} = \frac{74+70+66}{3} \approx 70$$

[0.5 mark]

$$\sigma_{M|P=Yes} = \sqrt{\frac{(4.67)^2+(-4.33)^2+(-0.33)^2}{3}} \approx \sqrt{13.56} \approx 3.682$$

$$\sigma_{M|P=No} = \sqrt{\frac{(4)^2+(0)^2+(-4)^2}{3}} \approx \sqrt{10.67} \approx 3.266$$

(c) Derive the required conditional probabilities of the contrinuous attributes given the outcome, using the probability density function for the normal distribution. **(4)**

Solution:

[0.5 mark]

$$\mathbb{P}(S = 6 \mid P = Yes) = \frac{1}{(3.559)\sqrt{2\pi}} \cdot e^{-\frac{(6-11)^2}{2\times 12.67}} \approx 0.04179$$

[0.5 mark]

$$\mathbb{P}(S = 6 \mid P = No) = \frac{1}{(2.944)\sqrt{2\pi}} \cdot e^{-\frac{(6-9)^2}{2\times 8.67}} \approx 0.08064$$

[0.5 mark]

$$\mathbb{P}(S = 9 \mid P = Yes) = \frac{1}{(3.559)\sqrt{2\pi}} \cdot e^{-\frac{(9-11)^2}{2\times 12.67}} \approx 0.09573$$

[0.5 mark]

$$\mathbb{P}(S = 9 \mid P = No) = \frac{1}{(2.944)\sqrt{2\pi}} \cdot e^{-\frac{(9-9)^2}{2\times 8.67}} \approx 0.13551$$

[0.5 mark]

$$\mathbb{P}(M = 75 \mid P = Yes) = \frac{1}{(3.682)\sqrt{2\pi}} \cdot e^{-\frac{(75-87.33)^2}{2\times 13.56}} \approx 0.000398$$

[0.5 mark]

$$\mathbb{P}(M = 75 \mid P = No) = \frac{1}{(3.266)\sqrt{2\pi}} \cdot e^{-\frac{(75-70)^2}{2\times 10.67}} \approx 0.03785$$

[0.5 mark]

$$\mathbb{P}(M = 82 \mid P = Yes) = \frac{1}{(3.682)\sqrt{2\pi}} \cdot e^{-\frac{(82-87.33)^2}{2\times 13.56}} \approx 0.03801$$

[0.5 mark]

$$\mathbb{P}(M = 82 \mid P = No) = \frac{1}{(3.266)\sqrt{2\pi}} \cdot e^{-\frac{(82-70)^2}{2\times 10.67}} \approx 0.000143$$

(d) Derive the ratio of required conditional probabilities for the outcome ($P = Yes/No$) for the given two new student data, namely X_1 and X_2, and indicate the outcome that a Naïve Bayes classifier would predict for these two new students? Show your calculations. **(2)**

Solution:

$$\frac{\mathbb{P}\Big(P = Yes \mid X_1 : \langle C = Theory,\ A = Average,\ S = 6,\ M = 75\rangle\Big)}{\mathbb{P}\Big(P = No \mid X_1 : \langle C = Theory,\ A = Average,\ S = 6,\ M = 75\rangle\Big)}$$

$$= \frac{\mathbb{P}(P = Yes)\mathbb{P}(C = Theory \mid P = Yes)\mathbb{P}(A = Average \mid P = Yes)\mathbb{P}(S = 6 \mid P = Yes)\mathbb{P}(M = 75 \mid P = Yes)}{\mathbb{P}(P = No)\mathbb{P}(C = Theory \mid P = No)\mathbb{P}(A = Average \mid P = No)\mathbb{P}(S = 6 \mid P = No)\mathbb{P}(M = 75 \mid P = No)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{3} \times \frac{1}{3} \times 0.04179 \times 0.000398}{\frac{1}{2} \times \frac{2}{3} \times \frac{1}{3} \times 0.08064 \times 0.03785} \quad \approx \quad 0.002725 \ < \ 1$$

Therefore, student X_1 will not be placed. [1 mark]

$$\frac{\mathbb{P}\Big(P = Yes \mid X_2 : \langle C = Lab,\ A = Low,\ S = 9,\ M = 82\rangle\Big)}{\mathbb{P}\Big(P = No \mid X_2 : \langle C = Lab,\ A = Low,\ S = 9,\ M = 82\rangle\Big)}$$

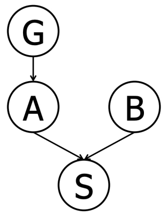
$$= \frac{\mathbb{P}(P = Yes)\mathbb{P}(C = Lab \mid P = Yes)\mathbb{P}(A = Low \mid P = Yes)\mathbb{P}(S = 9 \mid P = Yes)\mathbb{P}(M = 82 \mid P = Yes)}{\mathbb{P}(P = No)\mathbb{P}(C = Lab \mid P = No)\mathbb{P}(A = Low \mid P = No)\mathbb{P}(S = 9 \mid P = No)\mathbb{P}(M = 82 \mid P = No)}$$

$$= \frac{\frac{1}{2} \times \frac{2}{3} \times \frac{1}{3} \times 0.09573 \times 0.03801}{\frac{1}{2} \times \frac{1}{3} \times \frac{1}{3} \times 0.13551 \times 0.000143} \quad \approx \quad 383.59818 \ > \ 1$$

Therefore, student X_2 will be placed. [1 mark]

Q4. **[Bayesian Networks]** **10 marks**

Suppose that a patient can have a symptom (S) that can be caused by two different diseases (A and B). It is known that the variation of gene G plays a big role in the manifestation of disease A. The Bayes' Net and corresponding conditional probability tables for this situation are shown on right. The conditional and marginal probability values are given as,



$$\mathbb{P}(G) = 0.1, \quad \mathbb{P}(B) = 0.4, \quad \mathbb{P}(A \mid G) = 1.0, \quad \mathbb{P}(A \mid \neg G) = 0.1,$$
$$\mathbb{P}(S \mid A, B) = 1.0, \quad \mathbb{P}(S \mid A, \neg B) = 0.9, \quad \mathbb{P}(S \mid \neg A, B) = 0.8, \quad \mathbb{P}(S \mid \neg A, \neg B) = 1.0.$$

Answer the following. Show your calculations.

(a) Compute the probability of the joint disribution, that is, $\mathbb{P}(G, A, B, S)$. **(2)**

Solution:

$$\mathbb{P}(G, A, B, S) = \mathbb{P}(G)\, \mathbb{P}(A \mid G)\, \mathbb{P}(B)\, \mathbb{P}(S \mid A, B) = 0.1 \times 1.0 \times 0.4 \times 1.0 = 0.04$$

(b) Find the probability that a patient has disease A. **(2)**

Solution:

$$\mathbb{P}(A) = \mathbb{P}(A \mid G)\, \mathbb{P}(G) + \mathbb{P}(A \mid \neg G)\, \mathbb{P}(\neg G) = 1.0 \times 0.1 + 0.1 \times 0.9 = 0.19$$

(c) Find the probability that a patient has disease A given that they have disease B. **(1)**

Solution:

It can be infered from the graph of Bayes' Net that $A \perp B$. Therefore, we get:

$$\mathbb{P}(A \mid B) \;=\; \mathbb{P}(A) \;=\; 0.19$$

(d) Find the probability that a patient has disease A given that they have symptom S and disease B. **(2)**

Solution:

$$\begin{aligned}\mathbb{P}(A \mid S,B) &= \frac{\mathbb{P}(A,B,S)}{\mathbb{P}(A,B,S)+\mathbb{P}(\neg A,B,S)} = \frac{\mathbb{P}(A)\,\mathbb{P}(B)\,\mathbb{P}(S \mid A,B)}{\mathbb{P}(A)\,\mathbb{P}(B)\,\mathbb{P}(S \mid A,B)+\mathbb{P}(\neg A)\,\mathbb{P}(B)\,\mathbb{P}(S \mid \neg A,B)} \\ &= \frac{0.19 \times 0.4 \times 1.0}{0.19 \times 0.4 \times 1.0 + 0.81 \times 0.4 \times 0.8} = \frac{0.076}{0.076+0.2592} \approx 0.2267\end{aligned}$$

(e) Find the probability that a patient has the disease carrying gene variation G given that they have disease A. **(2)**

Solution:

$$\begin{aligned}\mathbb{P}(G \mid A) &= \frac{\mathbb{P}(G)\,\mathbb{P}(A \mid G)}{\mathbb{P}(G)\,\mathbb{P}(A \mid G) + \mathbb{P}(\neg G)\,\mathbb{P}(A \mid \neg G)} \\ &= \frac{0.1 \times 1.0}{0.1 \times 1.0 + 0.9 \times 0.1} = \frac{0.1}{0.1 + 0.09} \approx 0.5263\end{aligned}$$

(f) Find the probability that a patient has the disease carrying gene variation G given that they have disease B. **(1)**

Solution:

It can be infered from the graph of Bayes' Net that $B \perp G$. Therefore, we get:

$$\mathbb{P}(G \mid B) = \mathbb{P}(G) = 0.1$$

Q5. **[Classifier Evaluation]** **10 marks**

You are given with the following data set of 15 samples, $D = \{d_0, d_2, \ldots, d_{14}\}$ with known actual class labels, $L(d_i) \in \{A, B, C\}$ for each of the sample $d_i \in D$ $(0 \le i \le 14)$. In order to evaluate the quality of a classifier K, each sample $d_i \in D$ is additionally classified using K, yielding the predicted class label $K(d_i)$. The results are given in the table below.

$i \rightarrow$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Actual $\rightarrow L(d_i)$	A	B	A	C	C	B	A	A	A	B	B	C	C	C	B
Predicted $\rightarrow K(d_i)$	A	A	C	C	B	B	A	A	A	C	A	A	C	C	B

(a) Express the confusion matrix. **(2)**

Solution:

		Predicted Class $\#K(d_i)$		
		A	B	C
Actual Class $\#L(d_i)$	A	4	0	1
	B	2	2	1
	C	1	1	3

(b) Compute the accuracy. Show your calculations. **(2)**

Solution:

The accuracy is given by,

$$Accuracy(K) = \frac{|\{d \in D \mid K(d) = L(d)\}|}{|D|} = \frac{4+2+3}{15} = \frac{3}{5}$$

(c) Compute Precision, Recall and F_1-score for each class $c \in \{A, B, C\}$. Show your calculations. **(2×3)**

Solution:

Precision: [2 marks]

The precision is given by,

$$\begin{aligned} Precision(K,A) &= \frac{4}{4+2+1} = \frac{4}{7} \\ Precision(K,B) &= \frac{2}{0+2+1} = \frac{2}{3} \\ Precision(K,C) &= \frac{3}{1+1+3} = \frac{3}{5} \end{aligned}$$

Recall: [2 marks]

The recall is given by,

$$\begin{aligned} Recall(K,A) &= \frac{4}{4+0+1} = \frac{4}{5} \\ Recall(K,B) &= \frac{2}{2+2+1} = \frac{2}{5} \\ Recall(K,C) &= \frac{3}{1+1+3} = \frac{3}{5} \end{aligned}$$

F_1-score: [2 marks]

The F_1-score is given by,

$$\begin{aligned} F_1(K,A) &= \frac{2 \cdot Precision(K,A) \cdot Recall(K,A)}{Precision(K,A) + Recall(K,A)} = \frac{2 \times \frac{4}{7} \times \frac{4}{5}}{\frac{4}{7} + \frac{4}{5}} = \frac{2}{3} \\ F_1(K,B) &= \frac{2 \cdot Precision(K,B) \cdot Recall(K,B)}{Precision(K,B) + Recall(K,B)} = \frac{2 \times \frac{2}{3} \times \frac{2}{5}}{\frac{2}{3} + \frac{2}{5}} = \frac{1}{2} \\ F_1(K,C) &= \frac{2 \cdot Precision(K,C) \cdot Recall(K,C)}{Precision(K,C) + Recall(K,C)} = \frac{2 \times \frac{3}{5} \times \frac{3}{5}}{\frac{3}{5} + \frac{3}{5}} = \frac{3}{5} \end{aligned}$$

— Question Paper Ends Here —