CS60020 : Foundations of Algorithm Design and Machine Learning (Spring 2026)

Class Test 2

26-Mar-2026 (Wednesday) Maximum Marks: 30 06:30pm – 07:45pm

Roll: ____________________ **Name:** ____

[*Write your answers in question paper. Answer all questions. Be brief and precise.*]

Q1. [Shortest-Path in Graphs]

Present a weighted directed graph of at least 8 nodes, with all edge weights positive except exactly one negative edge, but no negative cost cycles, and show how Dijkstra's shortest cost path algorithm will fail on that example. Clearly show the working of Dijkstra's shortest cost path algorithm on the chosen example and explain how the algorithm fails. **(5)**

Solution:

To be given.

Q2. [Graph Traversal]

Given a weighted acyclic directed graph, a start vertex s, an end vertex g, an intermediate vertex n, answer in details the following questions:

(a) Present an efficient $O(V+E)$ algorithm to find the longest cost path *from s to g via n*. Clearly explain each step with comments justifying the working. **(7)**

(b) Derive the time complexity of your presented algorithm. **(3)**

Solution:

To be given.

Q3. [Logistic Regression]

For a binary logistic regression model with input attribute vector $\mathbf{x}$ and an output y, having an internal sigmoid activation function (of the form $\sigma(z) = \frac{1}{1+e^{-z}}$, where $z = \mathbf{w}^T\mathbf{x}$ with weight vector $\mathbf{w}$), we predict the output $\hat{y} = 1$ when $\mathbb{P}(y = 1 \mid \mathbf{x} \,;\, \mathbf{w}) \geq \frac{1}{2}$. Prove that this logistic regression model is also a linear classifier. **(4)**

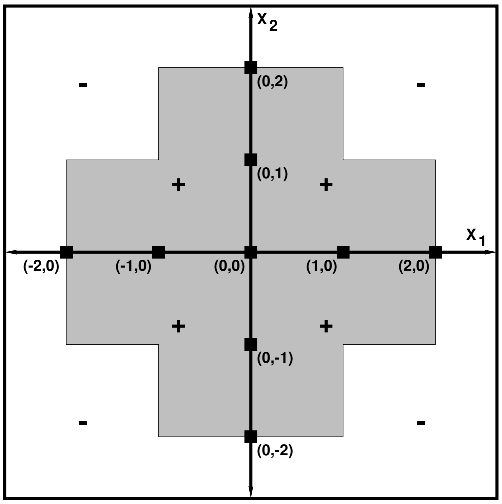
Solution:

$$\begin{aligned}
\mathbb{P}(y = 1 \mid \mathbf{x} \,;\, \mathbf{w}) \geq \frac{1}{2} \quad &\implies \quad \frac{1}{1+e^{-\mathbf{w}^T\mathbf{x}}} \geq \frac{1}{2} \\
&\implies \quad 1+e^{-\mathbf{w}^T\mathbf{x}} \leq 2 \\
&\implies \quad e^{-\mathbf{w}^T\mathbf{x}} \leq 1 \\
&\implies \quad -\mathbf{w}^T\mathbf{x} \leq 0 \\
&\implies \quad \mathbf{w}^T\mathbf{x} \geq 0
\end{aligned}$$

Therefore, we predict $\hat{y} = 1$ if $\mathbf{w}^T\mathbf{x} \geq 0$. [Proved]

Q4. [Artificial Neural Networks]

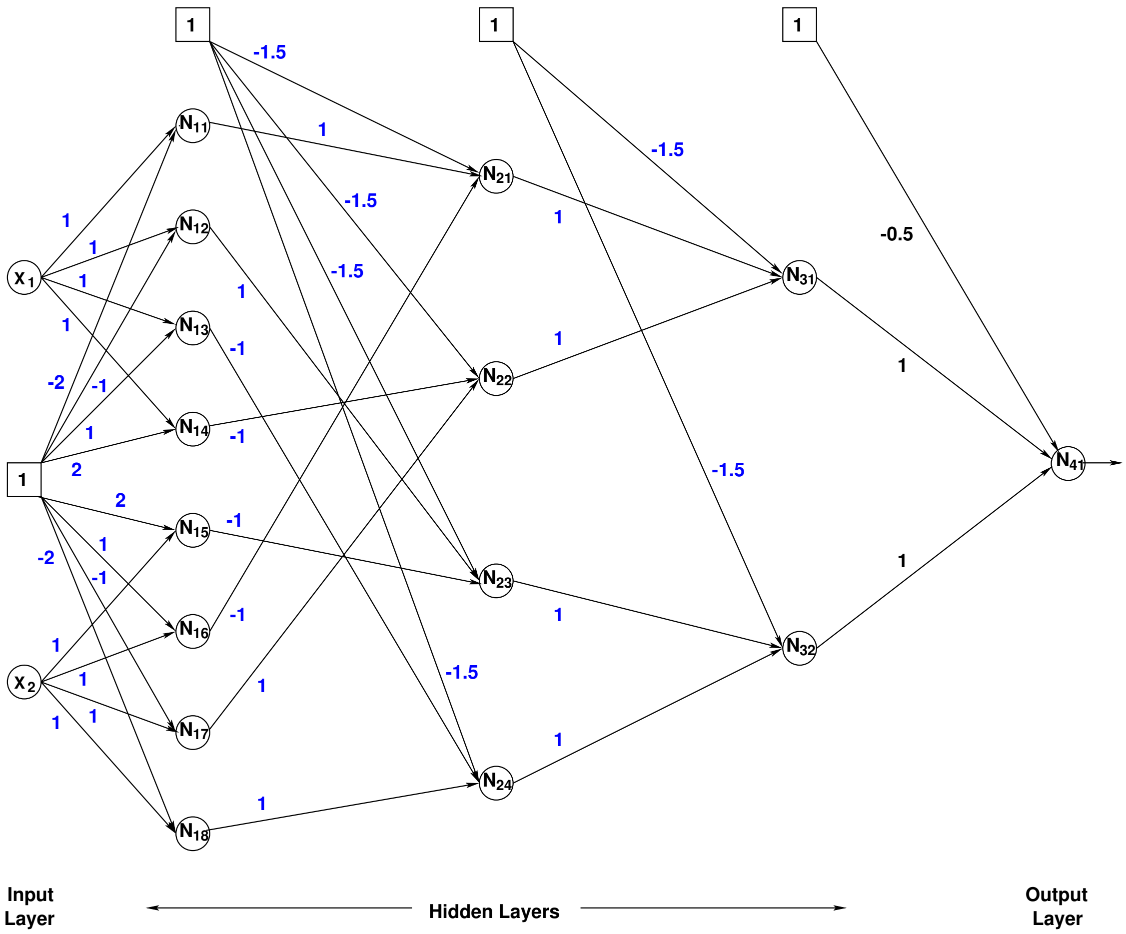
In this question, you need to create (not a fully-connected) *feed-forward artificial neural network* (ANN) which can classify every input, having attributes (x_1, x_2), that falls inside or outside of the shaded region as label '+' or '−', respectively (as shown in the right figure). For your reference, some input points (capturing (x_1, x_2) attribute pairs) are indicated using ■. *Assume that the nodes of the ANN are independent perceptrons.* Answer the following.



(a) Complete the following ANN model having one input, one output and three hidden layers (requiring 8, 4, and 2 nodes in the first, second and third hidden layer, respectively), by only indicating the missing weights for all edge provided below. *Note that the respective weights corresponding to the output layer node are already shown for reference.* **(5)**

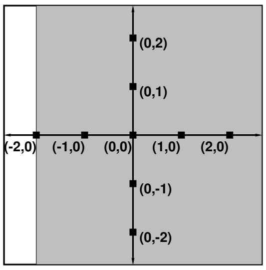
Solution:

Here, N_{11}, N_{12}, N_{13}, N_{14} accounts for classifying four vertical lines (required) where the right side of each of these lines are labeled as '+'; wereas N_{15}, N_{16}, N_{17}, N_{18} accounts for classifying four horizontal lines (required) wheret the above side of each of these lines are labeled '+'.

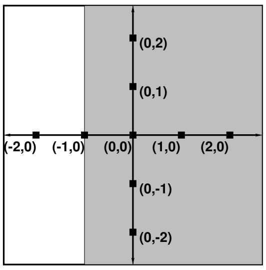


(b) Explain geometrically (and logically) what is being classified at every node of the hidden layer of the ANN (that you completed constructing) in part (a). For every representative node of the ANN specified in part (a), present the following: **(i)** For geometric representation, indicate '+' label by shaded region in the mentioned graphical boxes below; **(ii)** For logical representation, indicate the formula/rule in the blank provided below, using which such a classification is being made by the respective nodes. *Note that the respective entries corresponding to the output layer node are already shown as a reference (check the bottom-right graph and formula).* **(6)**

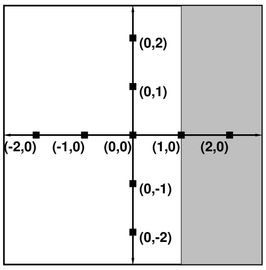
Solution:



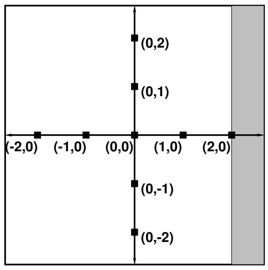
N_{11} : $x_1 \geq -2$



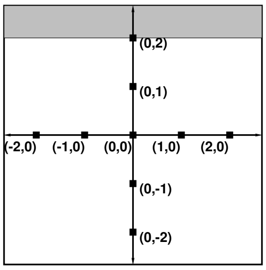
N_{12} : $x_1 \geq -1$



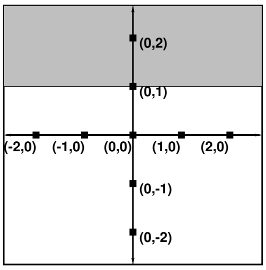
N_{13} : $x_1 \geq +1$



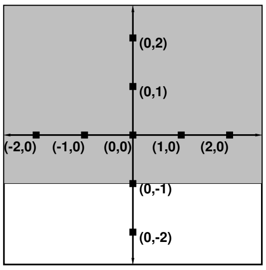
N_{14} : $x_1 \geq +2$



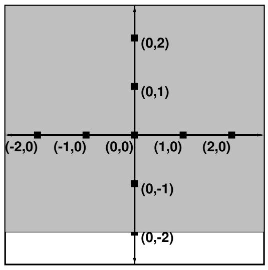
N_{15} : $x_2 \geq +2$



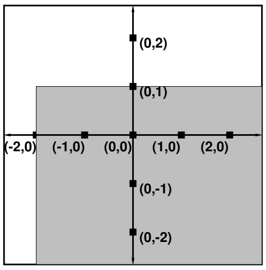
N_{16} : $x_2 \geq +1$



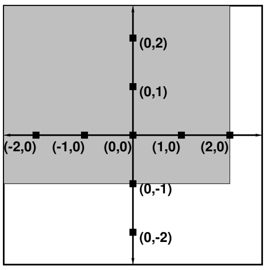
N_{17} : $x_2 \geq -1$



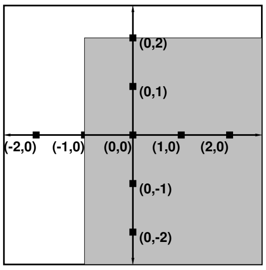
N_{18} : $x_2 \geq -2$



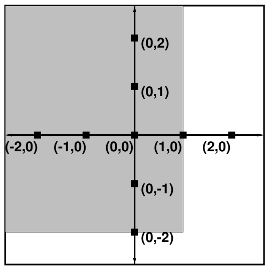
N_{21} : $N_{11} \wedge \overline{N_{16}}$



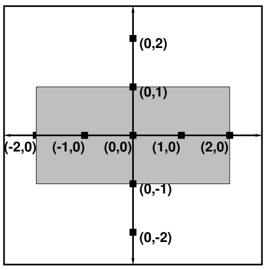
N_{22} : $\overline{N_{14}} \wedge N_{17}$



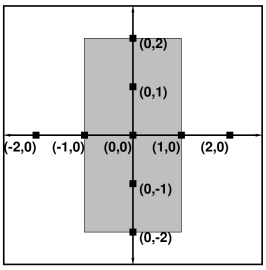
N_{23} : $N_{12} \wedge \overline{N_{15}}$



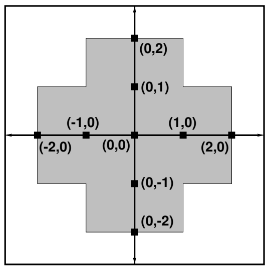
N_{24} : $\overline{N_{13}} \wedge N_{18}$



N_{31} : $N_{21} \wedge N_{22}$



N_{32} : $N_{23} \wedge N_{24}$



N_{41} : $N_{31} \vee N_{32}$

— End of Question Paper —