CS60020 : Foundations of Algorithm Design and Machine Learning (Spring 2026)

Class Test 1

28-Jan-2026 (Wednesday) Maximum Marks: 30 06:30pm – 07:45pm

[*Answer all questions. Be brief and precise. All parts of a question should be answered together.*]

Q1. [Algorithm Complexity]

Prove the following: If $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$, then $f(n) = \Theta(g(n))$. **(3)**

Q2. [Algorithm Design]

Consider the problem rod cutting, where given a rod of length R, and a set S of n pieces which form the required items that are needed to be cut (each piece less than R), you need to find a subset of S of pieces which can be cut from the rod R such that the remaining unusable portion of the rod is minimized. Please note that each cut also creates a wastage of size p. Therefore, we need to minimize the total loss comprising of the remaining unused portion and the wastage due to cuts.

Answer the following questions.

(a) Present a recursive definition to solve the problem. Clearly define the arguments, the return values, base condition, recursive calls and final solution formation. Explain each of the steps. **(8)**

(b) Show the working of your approach using a non-trivial example on a set S having at least 8 (eight) items. **(4)**

Q3. [Decision-Tree Learning]

Consider the following data set containing the information for 10 employees of IIT Kharagpur for a binary classification problem about their mode of travel to office. The Travel-Mode (by cycle or by car) has been collected along with their Designation (faculty or staff) and Residence (in campus or outside). For example, the first row of the following table is interpreted as – two *Faculty* members staying in *Campus* travel to their office via *Cycle*. Answer the following questions.

	(Attributes)		(Outcome)
# Instances	**Designation**	**Residence**	**Travel-Mode**
2	Faculty	Campus	Cycle
1	Faculty	Campus	Car
1	Faculty	Outside	Cycle
3	Faculty	Outside	Car
3	Staff	Campus	Cycle

(a) Calculate the overall entropy and Gini-index for this dataset (before any splitting). **(2)**

(b) Calculate the information gain when splitting on attributes, Designation and Residence. Which attribute would the decision tree algorithm choose to split? Show the calculations. **(4)**

(c) Calculate the Gini-index gain when splitting on attributes, Designation and Residence. Which attribute would the decision tree algorithm choose to split? Show the calculations. **(4)**

Q4. [Bayesian Learning]

Suppose $x_1, x_2, \ldots, x_n$ denote a set of random i.i.d. samples drawn from a Poisson distribution with mean $\lambda > 0$. The probability mass function of a Poisson is given as, $p(x \mid \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$. Derive the maximum likelihood estimator ($\hat{\lambda}$) for λ. **(5)**