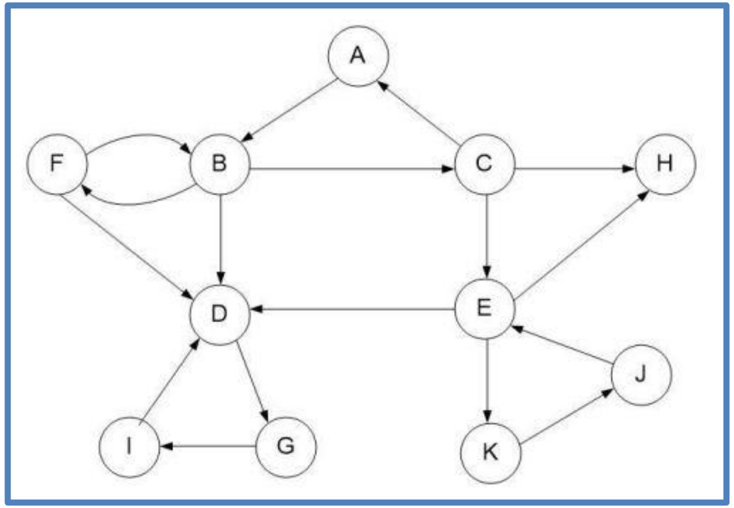
Traversal of Directed Graphs



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Directed Graphs

An Undirected Graph G = (V, E) consists of the following:

- A set of Vertices or Nodes V
- A set of DIRECTED Edges E where each edge connects two vertices of V. The edge is an ORDERED pair of vertices

Successor Function: succ(i) = {set of nodes to which node i is connected}

Directed Acyclic Graphs (DAGs): Such Graphs have no cycles (Figure 2)

Weighted Undirected Graphs: Such Graphs may have weights on edges (Figure 3). We can also have Weighted DAGs

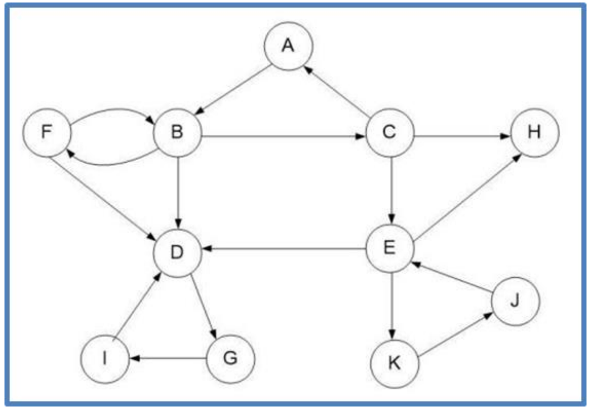


Figure 1

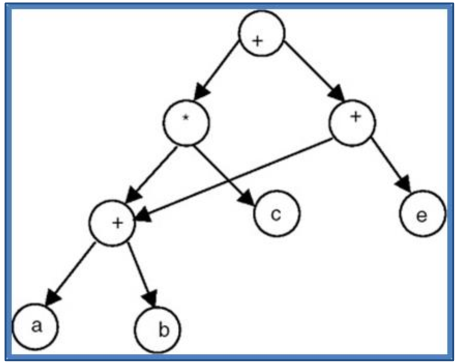


Figure 2

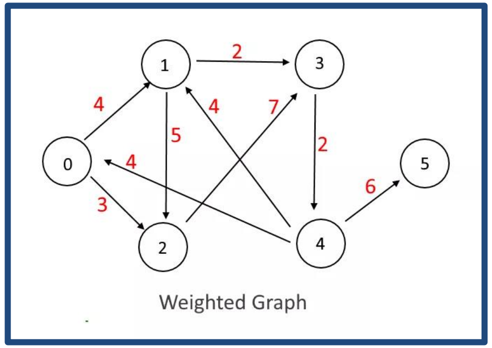
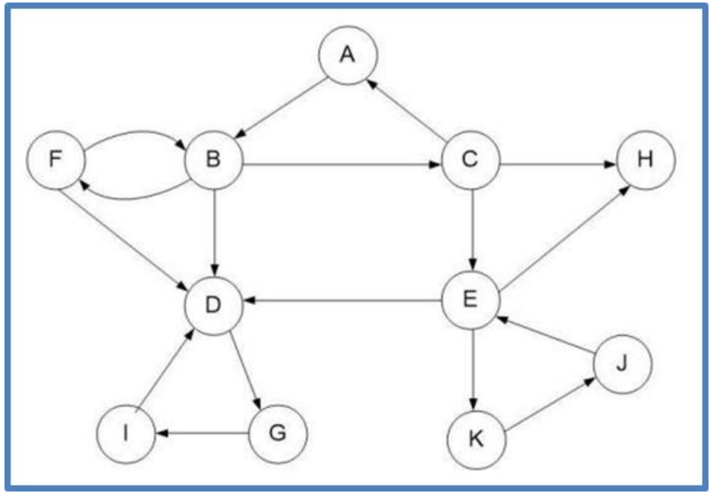


Figure 3

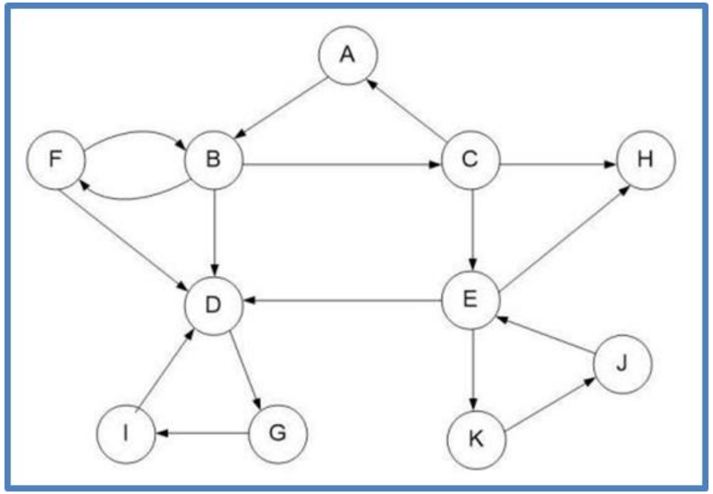
Basic Traversal Algorithm (Depth First Search)

```
Global Data: G = (V,E)
visited [i] indicates if node i is visited. / initially 0 /
Parent[i] = parent of a node in the Search / initially
NULL /
succ(i) = {set of nodes to which node i is
connected}
Dfs(node) {
    visited[node] = 1;
    for each j in succ(node) do {
        if (visited [j] ==0) { Parent[j] = node;
                               Dfs(j) }
    }
 }
```



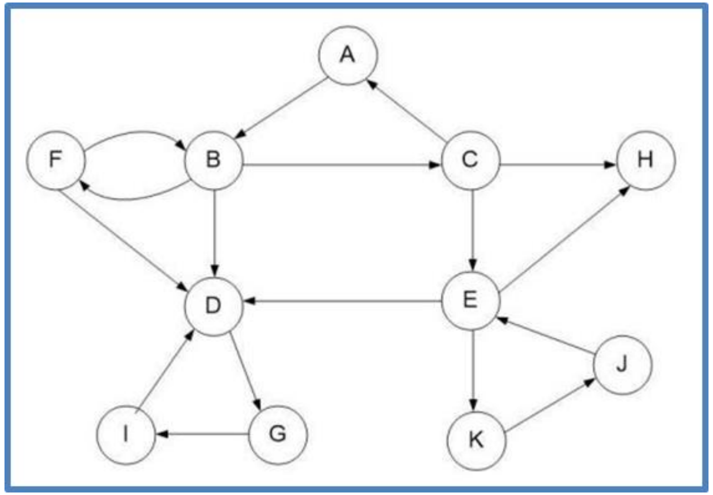
Traversing the Complete Graph by DFS

```
Global Data: G = (V,E)
visited [i] indicates if node i is visited. / initially 0 /
Parent[i] = parent of a node in the Search / initially
NULL /
succ(i) = {set of nodes to which node i is
connected}
Dfs(node) {
    visited[node] = 1;
    for each j in succ(node) do {
        if (visited [j] ==0) { Parent[j] = node;
                                         Dfs(j) }
     }
  }
```



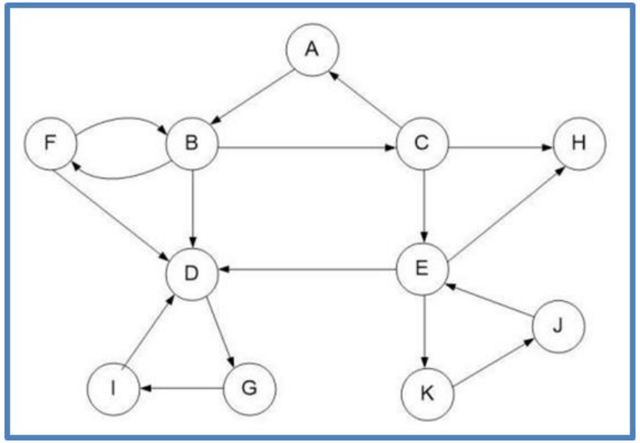
Entry-Exit Numbering

```
Global Data: G = (V,E)
visited [i] indicates if node i is visited. / initially 0 /
Parent[i] = parent of a node in the Search / initially
NULL /
Entry[i] = node entry sequence / initially 0 /
Exit[i] = node exit sequence / initially 0 /
succ(i) = {set of nodes to which node i is connected}
numb = 0;
Dfs(node) {
    visited[node] = 1; numb = numb+1;
    Entry[node] = numb;
    for each j in succ(node) do
        if (visited [j] ==0) { Parent[j] = node;
                               Dfs(j) }
     numb = numb + 1;
     Exit[node] = numb;
 }
```



Tree Edge, Back Edge, Forward Edge, Cross Edge

```
Global Data: G = (V,E)
visited [i] indicates if node i is visited. / initially 0 /
Parent[i] = parent of a node in the Search / initially NULL
/
Entry[i] = node entry sequence / initially 0 /
Exit[i] = node exit sequence / initially 0 /
succ(i) = {set of nodes to which node i is connected}
numb = 0;
Dfs(node) {
    visited[node] = 1; numb = numb+1;
    Entry[node] = numb;
    for each j in succ(node) do
        if (visited [j] ==0) { Parent[j] = node;
                               Dfs(j) }
     numb = numb + 1;
     Exit[node] = numb;
 }
```



Edge (u,v) is

Tree Edge or Forward Edge: if & only if

Entry[u] < Entry[v] < Exit[v] < Exit[u]

Back Edge: if & only if

Entry[v] < Entry [u] < Exit [u] < Exit [v]

Cross Edge: if & only if

Entry [v] < Exit [v] < Entry [u] < Exit [u]

Reachability, Paths, Cycles, Components

Global Data: G = (V,E)

visited [i] indicates if node i is visited. / initially 0 /

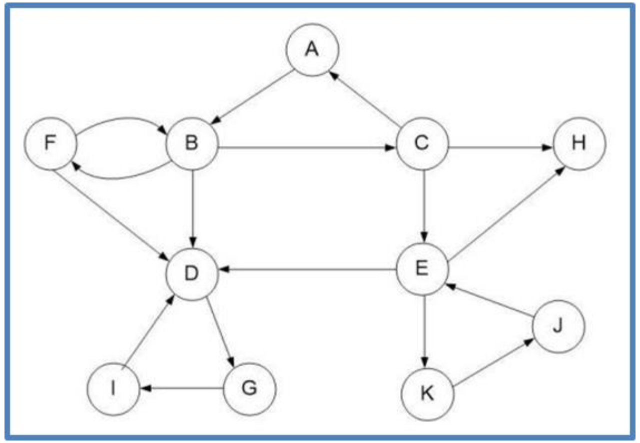
Parent[i] = parent of a node in the Search / initially NULL /

Entry[i] = node entry sequence / initially 0 /

Exit[i] = node exit sequence / initially 0 /

succ(i) = {set of nodes to which node i is connected}

```
numb = 0;
Dfs(node) {
    visited[node] = 1; numb = numb+1;
    Entry[node] = numb;
    for each j in succ(node) do
        if (visited [j] ==0) { Parent[j] = node;
                                Dfs(j) }
    numb = numb + 1;
    Exit[node] = numb;
}
```



Edge (u,v) is

Tree Edge or Forward Edge: if & only if

$Entry[u] < Entry[v] < Exit[v] < Exit[u]$

Back Edge: if & only if

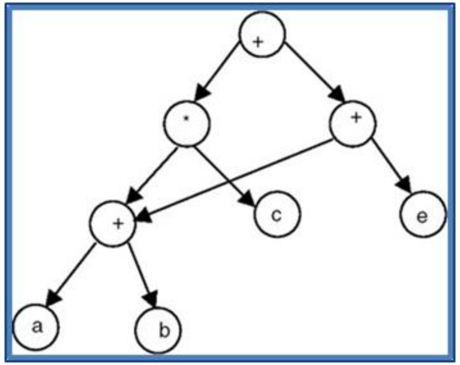
$Entry[v] < Entry [u] < Exit [u] < Exit [v]$

Cross Edge: if & only if

$Entry [v] < Exit [v] < Entry [u] < Exit [u]$

Directed Acyclic Graphs

```
Global Data: G = (V,E)
visited [i] indicates if node i is visited. / initially 0 /
Parent[i] = parent of a node in the Search / initially
NULL /
Entry[i] = node entry sequence / initially 0 /
Exit[i] = node exit sequence / initially 0 /
succ(i) = {set of nodes to which node i is connected}
numb = 0;
Dfs(node) {
    visited[node] = 1; numb = numb+1;
    Entry[node] = numb;
    for each j in succ(node) do
        if (visited [j] ==0) { Parent[j] = node;
                               Dfs(j) }
     numb = numb + 1;
     Exit[node] = numb;
 }
```



Topological Ordering, Level Values

Global Data: G = (V,E)

visited [i] indicates if node i is visited. / initially 0 /

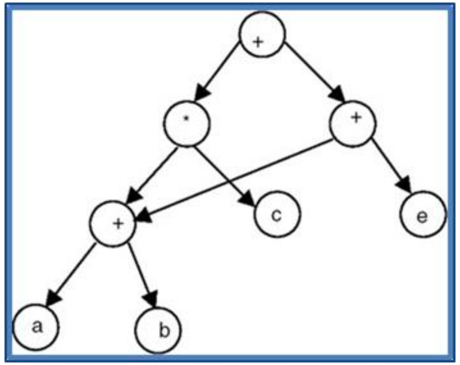
Parent[i] = parent of a node in the Search / initially NULL /

Entry[i] = node entry sequence / initially 0 /

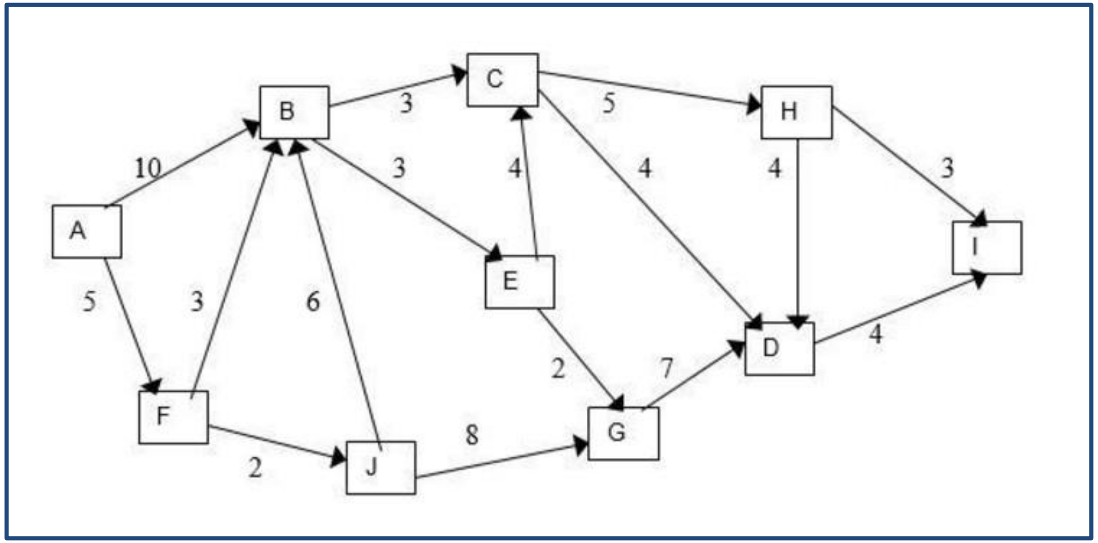
Exit[i] = node exit sequence / initially 0 /

succ(i) = {set of nodes to which node i is connected}

```
numb = 0; numb1 = 0;
Dfs(node) {
   visited[node] = 1; numb = numb+1;
   Entry[node] = numb;
   for each j in succ(node) do
      if (visited [j] ==0) { Parent[j] = node;
                             Dfs(j) }
    numb1 = numb1 + 1;
    Exit[node] = numb1;
 }
```



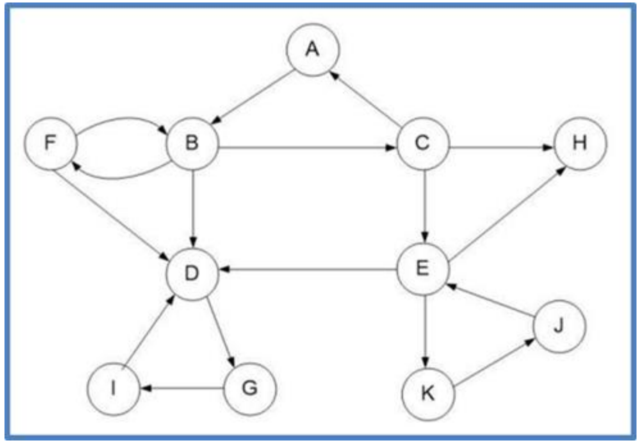
Shortest Cost Path in Weighted DAGs



Breadth-First Search

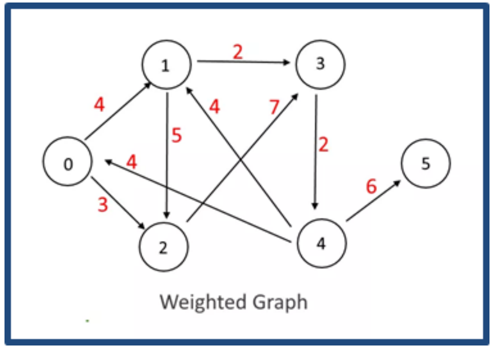
```
Global Data: G = (V,E)
Visited[i] all initialized to 0
Queue Q initially {}
 BFS(k) {
  visited [k] = 0;  Q = {k};
  While Q != {} {
    j = DeQueue (Q);
    if visited[j] == 0 {
    visited [j] = 1;
    For each k in succ (j) {
           if (visited[k]==0) EnQueue(Q,k); }
    }
    }
```

/Parent links, Shortest Length Path Finding in unweighted directed graphs/



Pathfinding in Weighted Directed Graphs

```
Global Data: G = (V,E)
Visited[i] all initialized to 0,
Cost[j] all initialized to INFINITY
Ordered Queue Q initially {}
 BFSW(k) {
 visited [k] = 0;  cost [k] = 0; Q = {k};
 While Q != {} {
   j = DeQueue (Q);
   if visited[j] == 0 {
   visited [j] = 1;
   For each k in succ (j) {
   if cost[k] > cost[j] + c[j,k]
             cost[k] = cost[j] + c[j,k];
   EnQueue(Q,k);}
  }
}
```



Thank you