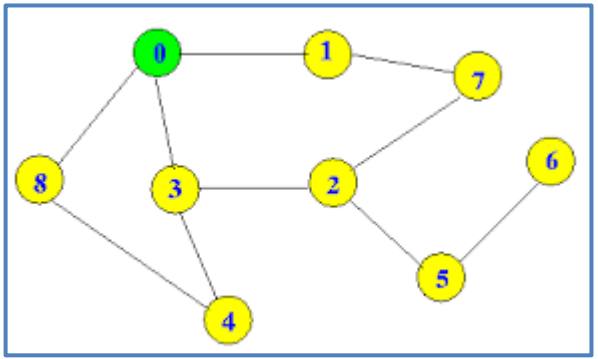
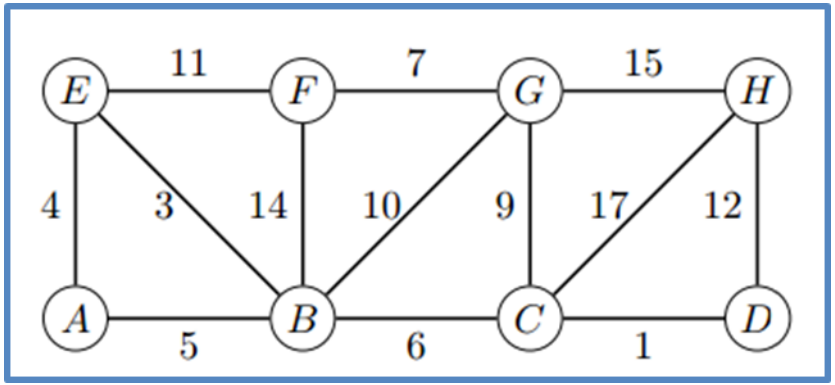
TRAVERSAL OF UNDIRECTED GRAPHS





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Undirected Graph

An Undirected Graph G = (V, E) consists of the following:

- A set of Vertices or Nodes V
- A set of Edges E where each edge connects two vertices of V

Example: Figure 1

V = {0,1,2,3,4,5,6,7,8}

E = {(0,1), (0,8), (0,3),(1,7), (2,3), (2,5), (2,7), (3,4), (4,8), (5,6)}

Successor Function: succ(i) = {set of nodes to which node i is connected}

Example: Succ(2) = {3,5,7}

Weighted Undirected Graphs: Such Graphs may have weights on edges (Figure 2)

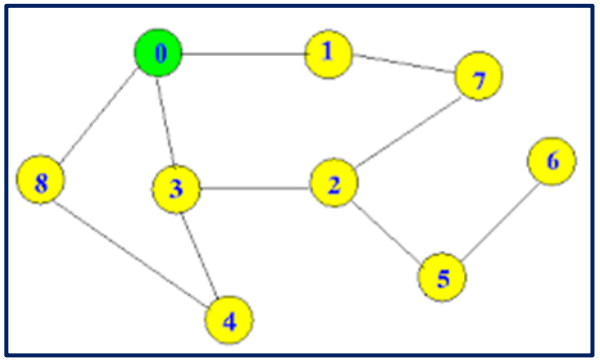


Figure 1

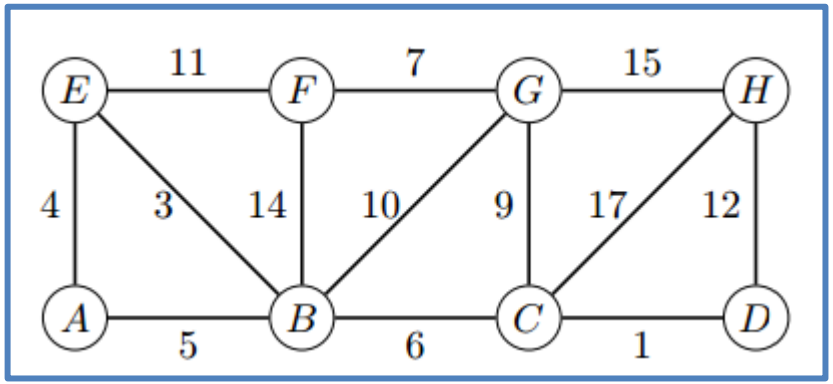


Figure 2

Problems on Undirected Graphs

Reachability, Path, Cycle / Tree Detection, Connected Components, Bi-Connected Components, Spanning Tree, Shortest Path, Maximum Flow, Vertex Cover, Edge Cover, Travelling Salesperson,

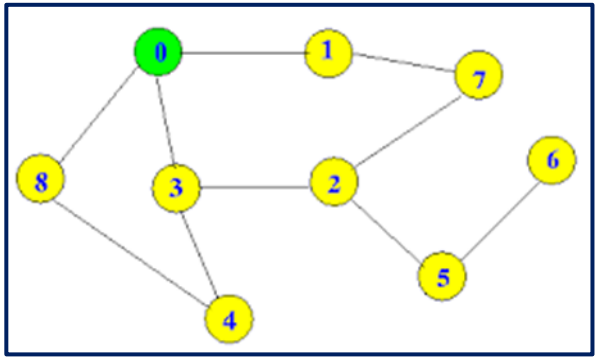
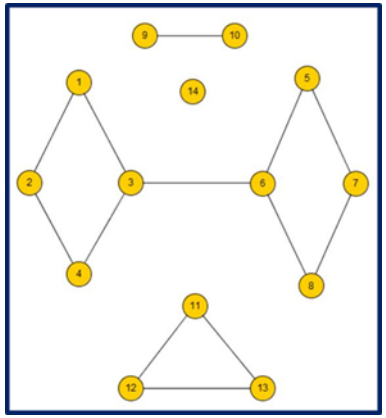


Figure 1



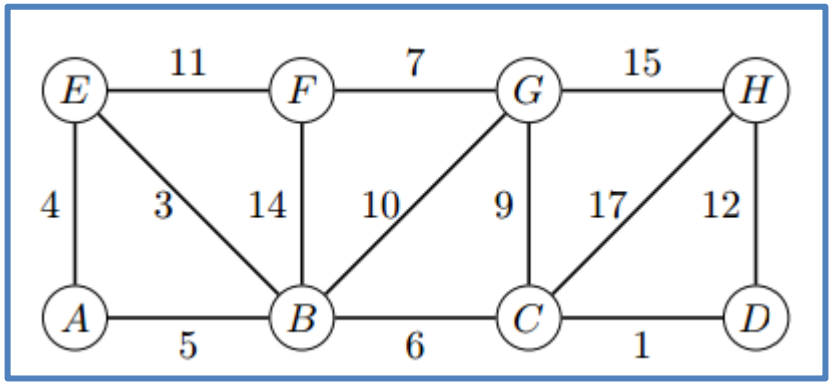


Figure 2

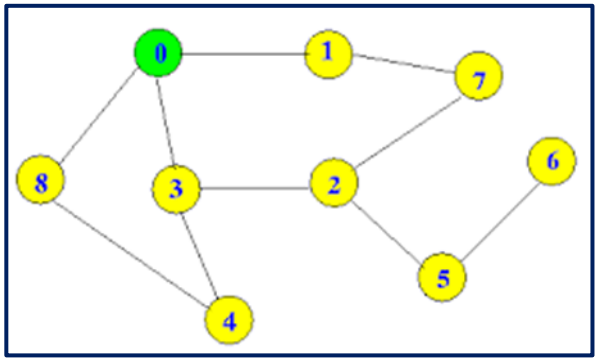
Basic Traversal Algorithm (Depth First Search) I

Global Data: G = (V,E)

visited [i] indicates if node i is visited. For all nodes j visited [j] is initialized to 0

succ(i) = {set of nodes to which node i is connected}

```
Dfs(node) {
   visited[node] = 1;
   for each j in succ(node) do {
      if (visited [j] ==0) Dfs(j)
    }
 }
```



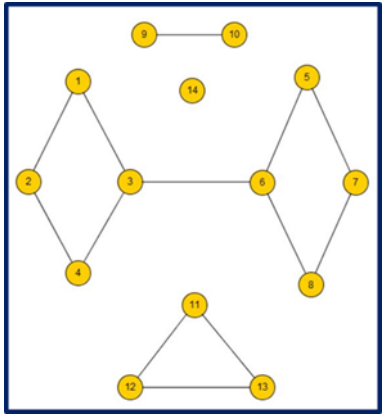
Basic Traversal Algorithm (Depth First Search) II

Global Data: G = (V,E)

visited [i] indicates if node i is visited. For all nodes j visited [j] is initialized to 0

succ(i) = {set of nodes to which node i is connected}

```
Dfs(node) {
   visited[node] = 1;
   for each j in succ(node) do {
      if (visited [j] ==0) Dfs(j)
    }
 }
```



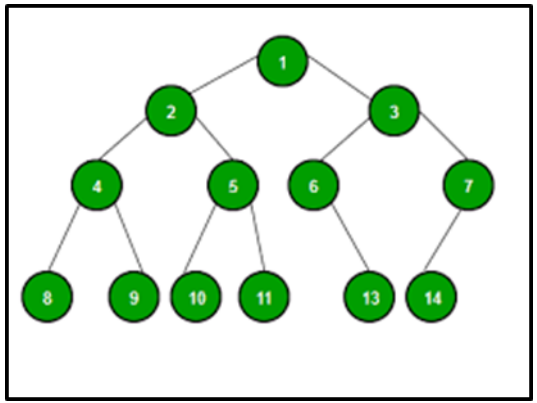
Basic Traversal Algorithm (Depth First Search) III

Global Data: G = (V,E)

visited [i] indicates if node i is visited. For all nodes j visited [j] is initialized to 0

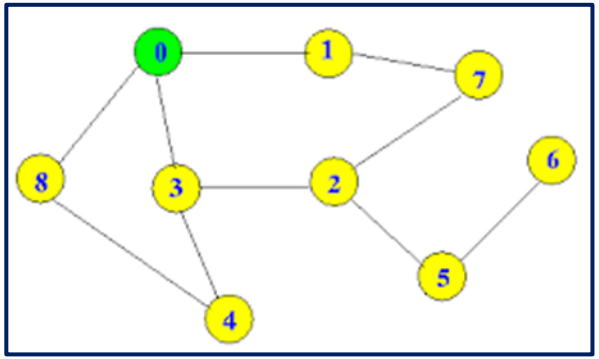
succ(i) = {set of nodes to which node i is connected}

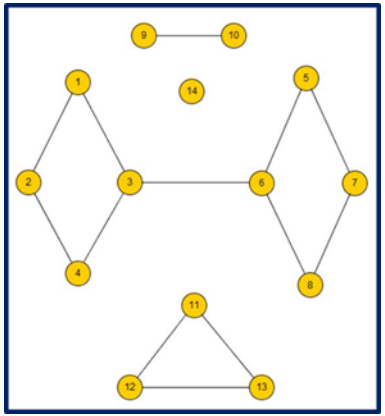
```
Dfs(node) {
   visited[node] = 1;
   for each j in succ(node) do {
      if (visited [j] ==0) Dfs(j)
    }
 }
```



Cycle Detection

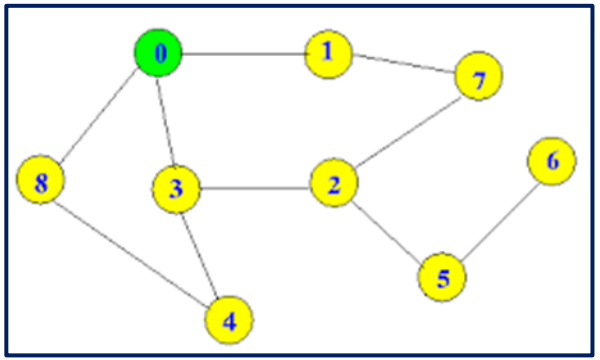
```
Global Data: G = (V,E)
visited [i] indicates if node i is visited. For all
nodes j visited [j] is initialized to 0
succ(i) = {set of nodes to which node i is
connected}
Dfs(node) {
   visited[node] = 1;
   for each j in succ(node) do {

         if (visited [j] ==0) Dfs(j)


    }
 }
// Cycle Detection //
```





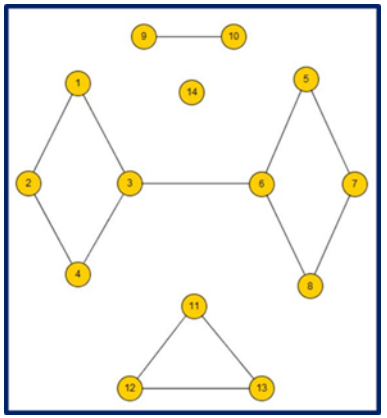
Path Finding

```
Global Data: G = (V,E)
visited [i] indicates if node i is visited. For all nodes j
visited [j] is initialized to 0

succ(i) = {set of nodes to which node i is connected}
Dfs(node) {
   visited[node] = 1;
   for each j in succ(node) do {
      if (visited [j] ==0) {

                Dfs(j) }
    }
 }
// Tree Edge, Back Edge, Parent Links, Tracing Paths
//
```



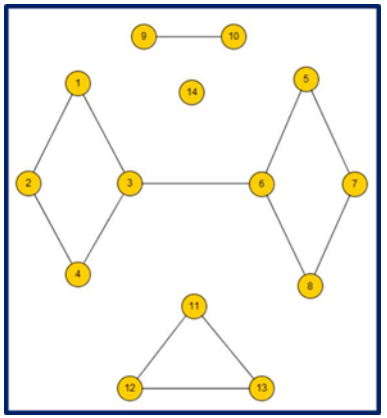
Connected Components

```
Global Data: G = (V,E)
Visited[i], comp[i] all initialized to 0
count = 0;

Algorithm components() {
for each node k do {
  if visited [k] == 0 { count = count + 1;
    DfComp_S(k) }

DfComp(node) {
   visited[node] = 1; comp[node] = count;
   for each j in succ(node) do {
      if (visited [j] ==0) DfComp(j)
    }
 }
```



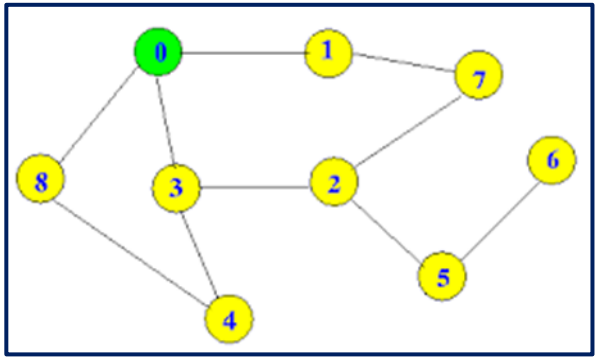
Depth-First Numbering & Time Stamping

```
Global Data: G = (V,E)
Visited[i], comp[i] all initialized to 0
count = 0;

Algorithm components() {
for each node k do {
  if visited [k] == 0 { count = count + 1;
    DfComp_S(k) }

DfComp(node) {
   visited[node] = 1; comp[node] = count;
   for each j in succ(node) do {
      if (visited [j] ==0) {

            DfComp(j)

          }
    }
 }
```



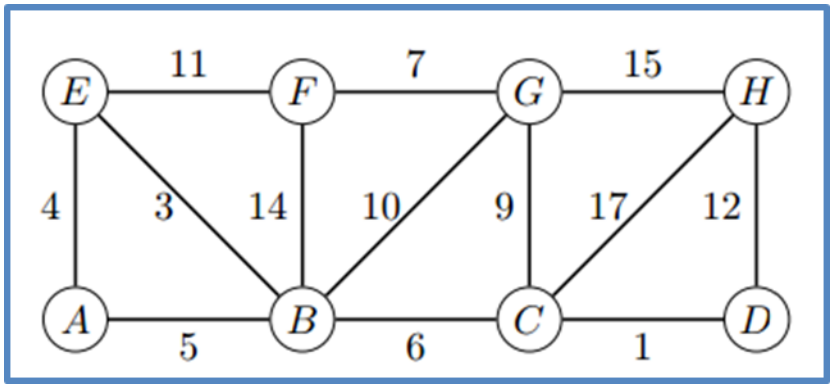
Breadth-First Search

```
Global Data: G = (V,E)
Visited[i] all initialized to 0
Queue Q initially {}
 BFS(k) {
  visited [k] = 0;  Q = {k};
  While Q != {} {
    j = DeQueue (Q);
    if visited[j] == 0 {
    visited [j] = 1;
    For each k in succ (j) EnQueue(Q,k);
    }
    }
/Parent links, Shortest Length Path Finding in
unweighted graphs/
```



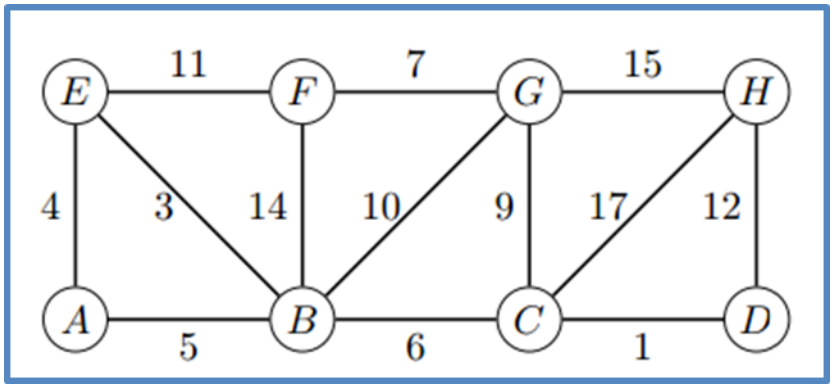
Pathfinding in Weighted Undirected Graphs I

```
Global Data: G = (V,E)
Visited[i] all initialized to 0,
Cost[j] all initialized to INFINITY
Ordered Queue Q initially {}
 BFSW(k) {
 visited [k] = 0;  cost [k] = 0; Q = {k};
 While Q != {} {
    j = DeQueue (Q);
    if visited[j] == 0 {
    visited [j] = 1;
    For each k in succ (j) {
    if cost[k] > cost[j] + c[j,k]
              cost[k] = cost[j] + c[j,k];
    EnQueue(Q,k);}
  }
}
```



Pathfinding in Weighted Undirected Graphs II

```
Global Data: G = (V,E)
Visited[i] all initialized to 0,
Cost[j] all initialized to INFINITY
Ordered Queue Q initially {}
 BFSW(k) {
 visited [k] = 0;  cost [k] = 0; Q = {k};
 While Q != {} {
    j = DeQueue (Q);
    if visited[j] == 0 {
    visited [j] = 1;
    For each k in succ (j) {
    if cost[k] > cost[j] + c[j,k]
            cost[k] = cost[j] + c[j,k];
    EnQueue(Q,k);}
  }
}
```



Thank you