Tutorial
Discrete Structures (CS21001)

Autumn Semester 2014

September 24, 2014

1. Consider the particular sequence defined by the initial conditions $a_0 = 4$ and $a_1 = 3$ and the recurrence relation $a_n = 5a_{n-1}+6a_{n-2}$ for $n = 2, 3, \ldots$ Let $G(x) = \sum_{n=0}^{\infty} a_n x^n$ be the generating function for this sequence. Derive an explicit formula for G(x) (as a rational function of x, e.g., $\frac{15x^2}{(1+x+x^2)}$ **directly from the given recurrence and initial conditions** (i.e., without using formulas for a_n derived by other methods).

2. The following questions are independent of each other.

   (a) Find the general solution to the recurrence $a_n = 8a_{n-1} - 16a_{n-2}$.

   (b) Find the general solution to the recurrence $a_n = 8a_{n-1} + 9a_{n-2}$.

   (c) Find a particular solution to the recurrence $a_n = 8a_{n-1}+9a_{n-2}+16n$. .

3. A computer system considers a string of decimal digits to be a valid code word if and only if it contains an even number of zero digits. For instance, 1230407869 is valid, whereas 3141529046 is not. Let V_n be the number of valid n-digit code words; find a recurrence for V_n .

4. How many ways are there to tile an $n \times 3$ board using 2×1 dominoes? Model this counting problem using a coupled set of recurrence relations which in theory could be solved to give you a closed formula.