TUTORIAL 4

1) Given the poset ({1, 2, 3, 5, 6, 7, 10, 20, 30, 60, 70}, |)
   a) Draw the Hasse Diagram for this poset.
   b) Find the maximal elements.
   c) Find the minimal elements.
   d) Find the greatest element
   e) Find the least element
   f) Find all upper bounds of {2, 5}
   g) Find the least upper bound of {2, 5} (if it exists)
   h) Find all lower bounds of {6, 10}
   i) Find the greatest lower bound of {6, 10} (if it exists)
   j) is this poset a lattice? Justify your answer

2) Let S = {x, y, z}, and consider the set P(S) with relation R given by set inclusion. Is R a partial order?

3) Prove that the direct product of any two distributive lattice is a distributive.

4) Prove that if l_1 and l_2 are elements of a lattice $< L; \vee , \wedge >$ then
   $(l_1 \vee l_2 = l_1) \leftrightarrow (l_1 \wedge l_2 = l_2) \leftrightarrow (l_1 \leq l_2)$

5) If $[L, \wedge , \vee]$ is a complemented and distributive lattice, then the complement of any element $a \in L$ is unique.

Hint:
Question 3: Let L_1 be the two element lattice with universe $\{x_0,x_1\}$, and $x_0<x_1$. We sometimes say $L_1\cong 2$. Let L_2 be another two element lattice with universe $\{y_0,y_1\}$, and $y_0<y_1$. Then $L_1\times L_2\cong 2\times 2$ is just the lattice whose Hasse diagram looks like a diamond. The top is the element (x_1,y_1). The bottom is (x_0,y_0). The other elements (x_0,y_1) and (x_1,y_0) are incomparable.