Solution Tutorial I
Discrete Structures
Autumn 2014

(a) Is this relation transitive and symmetric

{(1,2), (2,3), (1,3), (2,1)}

Sol. (1,1) and (2,2) are missing, No transitive
(3,2) and (3,1) are missing. No symmetric

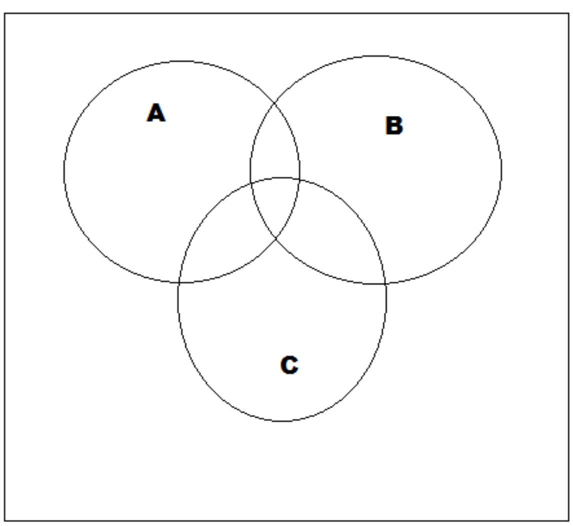
(b) Power set of a transitive set is transitive.

Proof. Assume X is transitive. Let $A \in B \in P(X)$. Since $B \in P(X)$, $B \subseteq X$. Thus, $A \in X$. Since X is transitive, $A \subseteq X$. Hence, $A \in P(X)$. It follows that P(X) is transitive.

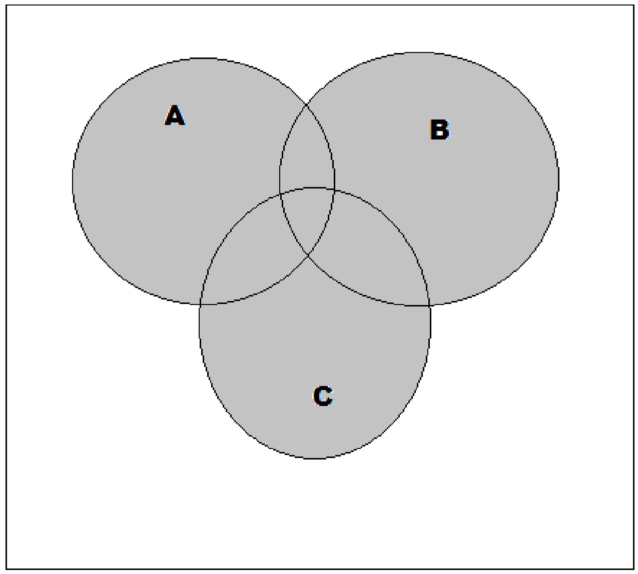
(c) Let A, B, and C be 3 events associated with a random experiment. Express the Following Verbal Statements both in set theory notation and by means of Venn diagrams:

(a) At least 1 of the events occurs;
(b) Exactly 1 of the events occurs;
(c) Exactly 2 of the events occur;
(d) Not more than 2 of the events occur simultaneously;
(e) All of the events occur;
(f) None of the events occur.

Sol.



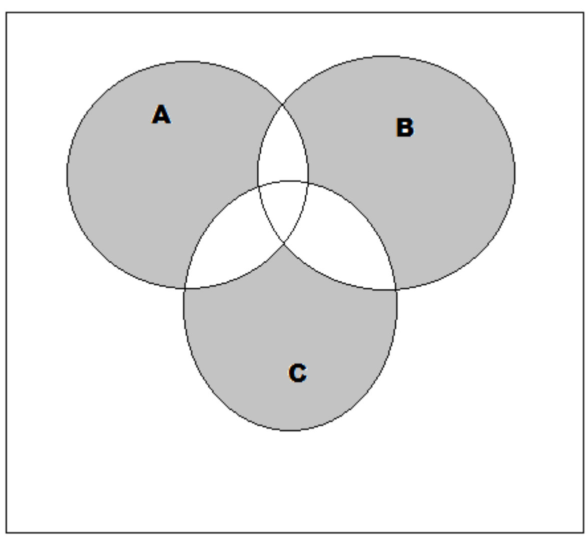
a) "At least one" is the complement of "none of them", i.e., $(A^c \cap B^c \cap C^c)^c$, which by De Morgan's law is also $A \cup B \cup C$. The Venn diagram is:



b) "Exactly one" means "A but not B and not C" or "B but not A and not C", or "C but not A and not B),

$$(A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C)$$

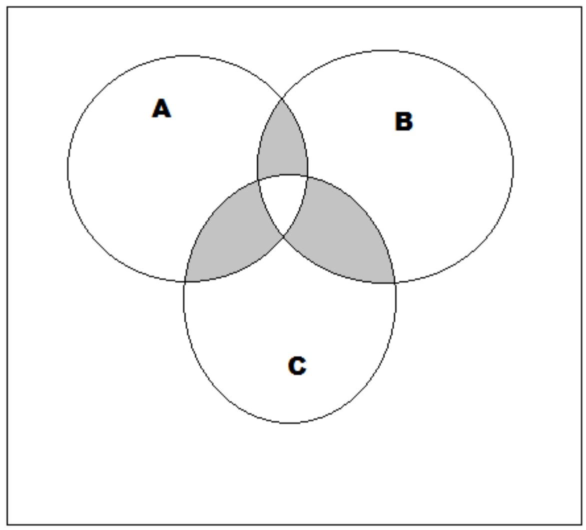
TheVenn diagram is:



c) “Exactly due” means “A and B but not C” or “A and C but not B”, or “B and C but not A”

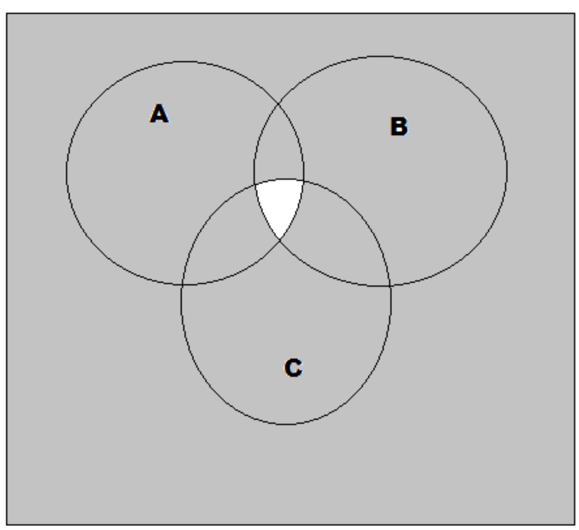
$$\mathbf{(A \cap B \cap C^c) \cup (A \cap B^c \cap C) \cup (A^c \cap B \cap C)}$$

Here’s the Venn diagram.



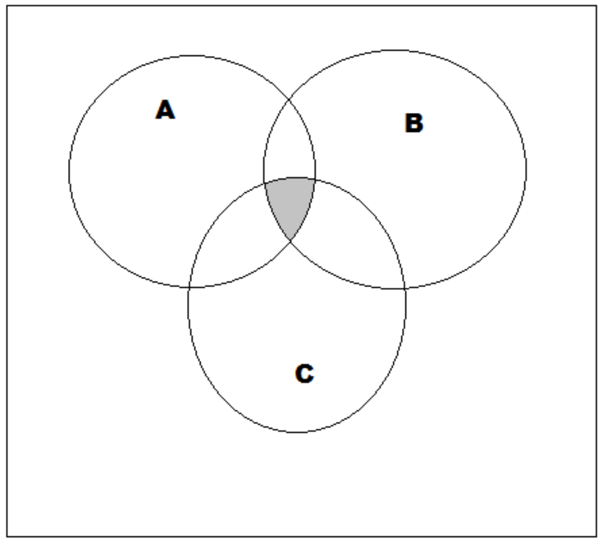
d) “Not more than 2”. The complement to this is: “More than 2”, i.e., all three events together, i.e. $(A \cap B \cap C)^c$

The Venn diagram is:

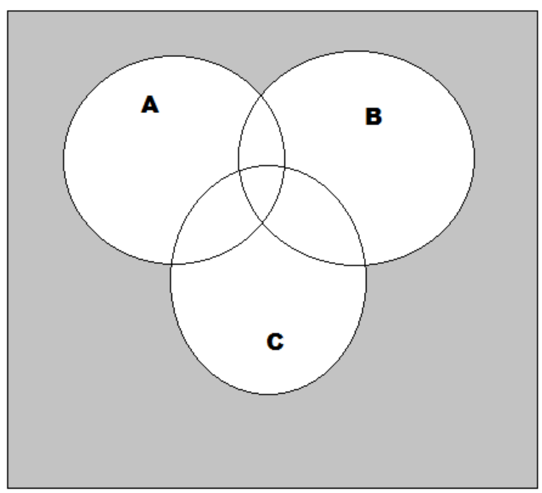


e) “All of them” is simply: $(A \cap B \cap C)$.

Here’s the Venn diagram



f) “None of them” is simply $(A^c \cap B^c \cap C^c)$, with Venn diagram:



d) Prove that for all sets A, B, and C,
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof:
(I) We first prove that $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$. Let $x \in A \cap (B \cup C)$. By definition of intersection, $x \in A$ and $x \in B \cup C$. Thus $x \in A$ and by definition of union, $x \in B$ or $x \in C$.
Case 1 ($x \in A$ and $x \in B$): In this case, by definition of intersection $x \in A \cap B$, and so by definition of union, $x \in (A \cap B) \cup (A \cap C)$.
Case 2 ($x \in A$ and $x \in C$): In this case, by definition of intersection $x \in A \cap C$, and so by definition of union, $x \in (A \cap B) \cup (A \cap C)$.
Hence in either case, $x \in (A \cap B) \cup (A \cap C)$.

(II) We now prove that $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$. Let $x \in (A \cap B) \cup (A \cap C)$. By definition of union, $x \in A \cap B$ or $x \in A \cap C$.
Case 1 ($x \in A \cap B$): In this case, by definition of intersection $x \in A$ and $x \in B$. Since $x \in B$, by definition of union, $x \in B \cup C$. Hence $x \in A$ and $x \in B \cup C$, and so by definition of intersection, $x \in A \cap (B \cup C)$.
Case 2 ($x \in A \cap C$): In this case, by definition of intersection and $x \in A$ and $x \in C$. Since, by definition of union, $x \in B \cup C$. Hence $x \in A$ and $x \in B \cup C$, and so by definition of intersection, $x \in x \in A \cap (B \cup C)$.
In either case $x \in A \cap (B \cup C)$.

e) Give an example of a relation which is not reflexive, not symmetric, not anti symmetric, and not transitive on a set {1,2,3}
sol. R = {(1, 2), (2, 1), (2, 3)}