

## TUTORIAL QUIZ II

- 1) One day Akbar asked birbal to prove that he is the wisest man in his kingdom. Birbal thought it must be Akbar's trick so he challenged Akbar to give him a sequence of  $n$  randomly selected numbers arranged in any order which Akbar wants to and then he (Birbal) would find indices  $i$  and  $j$  such that the sum of numbers between  $i$  and  $j$  would be divisible by  $n$ . Akbar made an announcement that whoever can come up with such a sequence will be rewarded heavily. Can Akbar win? Justify your answer.

(Note-If yes, then give such a sequence which satisfies the above condition....)

- 2) Let  $\tau \in S_n$  (symmetric group), and write  $\tau$  as a product of disjoint cycles (including 1-cycles):

$$\tau = \tau_1 \tau_2 \cdots \tau_m.$$

Let  $(x_1 x_2 \cdots x_k)$  be a  $k$ -cycle in  $S_n$ . Prove that for any  $\sigma \in S_n$ ,

$$\sigma(x_1 x_2 \cdots x_k)\sigma^{-1} = (\sigma(x_1) \sigma(x_2) \cdots \sigma(x_k)).$$

Where, the symmetric group  $S_n$  on a finite set of  $n$  symbols is the group whose elements are all the permutations of the  $n$  symbols.

- 3) Let,  $h$  be a group homomorphism from a group  $G$  to a group  $H$ . We define kernel of  $h$  as follows

$$\ker(h) := \{u \in G : h(u) = e_H\}.$$

**Prove that  $h$  is one-to-one if and only if  $\text{Ker}(h) = \{e\}$ .**