Tutorial Quiz I

Each Question carry 5 marks

1) Show that every nonempty finite subset of a lattice has a least upper bound and a greatest lower bound.

2) Let $A \subseteq Z$ and $f: A \rightarrow N$ be a one-one function where Z is a set of integers and N is a set of natural numbers. Let R be a relation on A defined as under:

$(x,y) \in R$ if and only if $f(y) = k\, f(x)$ where $k \in N$

Prove that R is a partial order relation on A.

3) Recall that the Fibonacci numbers are defined by $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-2} + f_{n-1}$ for $n \geq 2$. Prove that

$$f_0 f_1 + f_1 f_2 + \cdots + f_{2n-1} f_{2n} = f^2_{2n}$$

for any positive integer n.

Bonus Question:-

There are 6 thieves of IIT KGP. They unearth a long lost treasure which contains 60 precious stones.
There is an order of seniority amongst the thieves (eg. A, B ,C, D, E, F) . A is the most senior whereas
F is the junior most amongst them. Now they need to distribute the treasure amongst themselves. There is a rule that if a majority of the people support the distribution then the distribution is accepted else if a majority does not approve then the person is thrown out of the group(can also be killed) and then the next person proposes a distribution (Majority means ≥ 50% of the votes including his vote) .
What distribution must A put forward so that he maximizes his profits and also does not get killed !!!!!!
Note: Since all of them are KGPians they are very smart and very cunning.
A needs atleast 3 votes to survive.