

Name :

Roll no. :

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1. Answer all questions.
 2. All parts of a particular question should be answered together.
 3. Credits will be given for neat and to-the-point answering.
 4. Unnecessary / confusing words are liable to negative marking.
 5. If answer is copied from any source, it will be awarded zero marks.
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1. Consider a random walk on an connected, non-bipartite, undirected graph G . Show that, in the long run, the walk will traverse each edge with equal probability. (5)

2. A run is a sequence of coin tosses with the same outcome (for example, the sequence $HHHHTTHTTTTHH$ has 5 runs). Now suppose we toss a fair coin n times. What is the expected number of runs? (5)

3. **[Bonus Question]**

(5)

A mixed graph is graph in which some of the edges are directed and some of the edges are undirected. If a given mixed graph G has no directed cycle, then it is always possible to orient the remaining undirected edges so that the resulting graph has no directed cycle. Give an efficient algorithm for obtaining such an orientation if one exists. Prove the correctness of your algorithm and analyze its complexity.



4. **[Bonus Question]**

(5)

Given n vertices v_1, \dots, v_n , a random graph is formed as follows: for each pair of vertices v_i, v_j with $i \neq j$ we flip a fair coin, and include directed edge (v_i, v_j) in the graph if and only if the coin comes up heads. Two labeled graphs are identical if they have the same set of edges. Assume that the graphs are represented as adjacency matrices. What is the probability that two random graphs G_1 and G_2 formed on the same n vertices v_1, \dots, v_n are identical? Justify your answer.

