

Name :

Roll no. :

- 
1. Answer all questions.
  2. All parts of a particular question should be answered together.
  3. Credits will be given for neat and to-the-point answering.
  4. Unnecessary / confusing words are liable to negative marking.
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1. For line equation  $y = \frac{c}{b} - \frac{a}{b}x$ , prove that the line has integer solution for  $x, y$  ***if and only if***  $\gcd(a, b)$  divides  $c$ . (6)


2. Suppose  $x, y$  and  $a$  are non-zero integers.

(8)

1.  $d$  is the minimum value of  $px+qy$  where  $p, q$  are integers and  $px+qy > 0$ .

2.  $e$  is the minimum value of  $kx+l(ax+y)$  where  $k, l$  are integers and  $kx+l(ax+y) > 0$ .

Prove that  $d$  and  $e$  are equal.

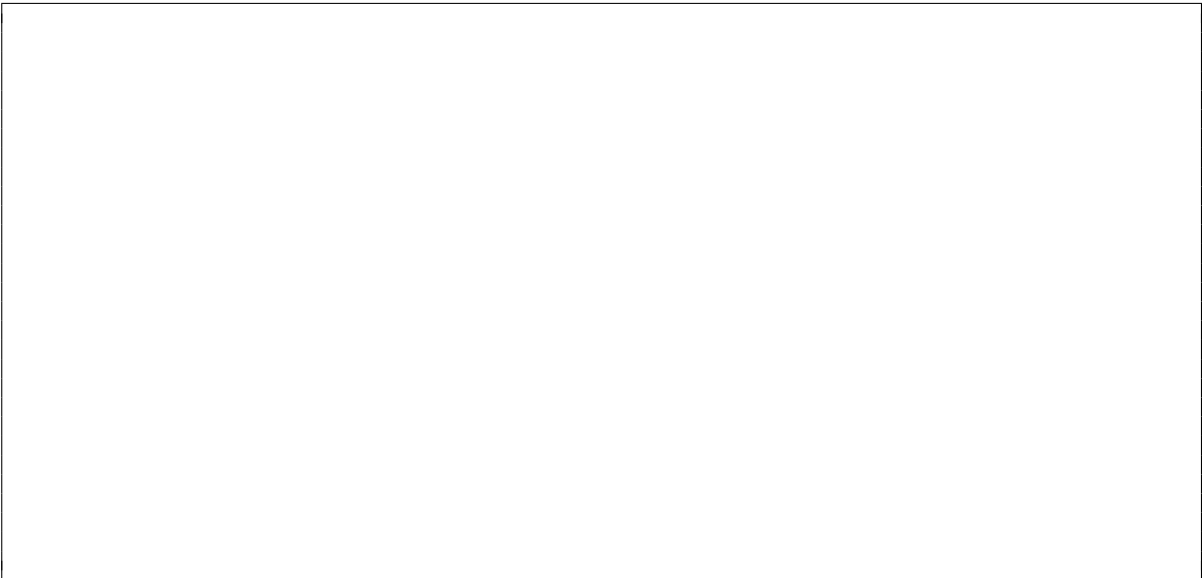


3. **[Bonus Question]**

(5)

Prove that if  $x|y$  and  $x|z$  then  $x|(yp + zq)$  for every  $p$  and  $q$  are integers.

(Note:- Here  $x|y$  means "**x divides y**")



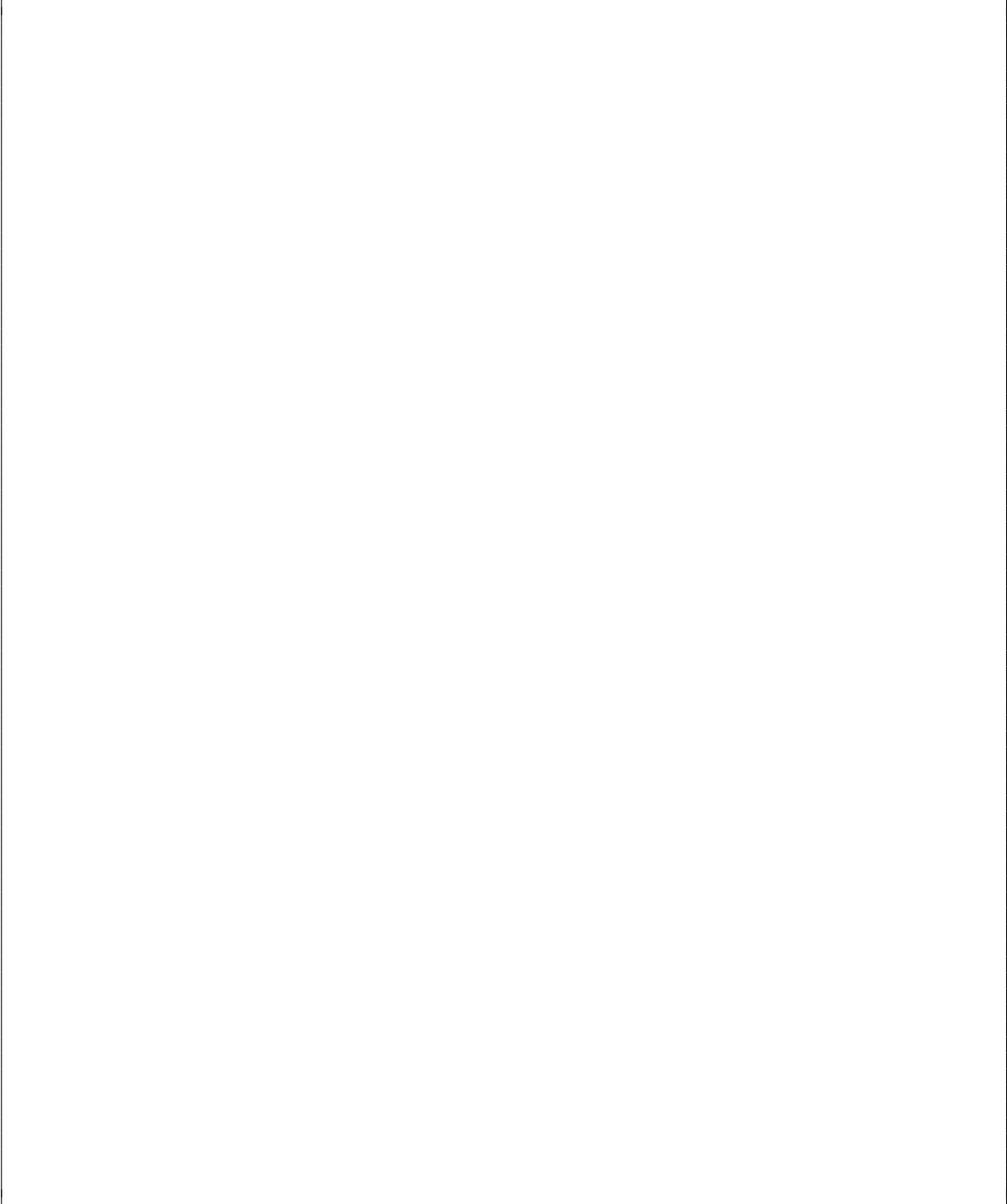
4. Suppose you are given a map with the x and y coordinates of your town with N houses. You have to find the number of line segments that will contain 4 or more houses. Give an efficient algorithm for this task and analyze the time complexity.  
NOTE: Hashing and similar functions are not allowed.

(10)

5. You are the TA for the DAA course in IIT Kharagpur. You are given the midsem and endsem marks for the  $N$  Students in the course. A student  $P$  is said to dominate a student  $Q$ , if the midsem and endsem marks of  $P$  are both greater than the respective midsem and endsem marks of  $Q$ . Design an efficient algorithm for finding all the students that are not dominated by any other student in the class. Analyze the time complexity as well.

(10)

NOTE: Hashing and similar functions are not allowed.



6. **Bonus Question**

Let  $C$  be a convex polygon with  $n$  vertices  $(V_1, V_2, \dots, V_n, V_1)$  going counterclockwise around the polygon. Let a direction be given by a vector  $u$ . We want to find the extreme, maximum and minimum, vertices of  $C$  in the direction  $u$ . That is, if the vertices  $V_i$  are orthogonally projected onto a line  $L$  in the direction  $u$ , then the extreme vertices are the ones whose projections are the extreme points on  $L$ . Write an efficient algorithm to find these extreme points and analyze the time complexity.

