

Name :

Roll no. :

1. Answer all questions.
2. All parts of a particular question should be answered together.
3. Credits will be given for neat and to-the-point answering.
4. **Unnecessary / confusing words** are liable to negative marking.
5. **Please attempt only three questions from the first four questions.** Total 5 question to attempt.

1. There are  $n$  boat trading shops numbered from 1 to  $n$ , along the coast of a river. As shown in Figure 1. Initially you are at shop number 1 and your goal is to reach shop number  $n$ . At any shop  $i$  you can rent a boat which has to be returned at shop  $j$  where  $j > i$  and  $j \leq n$ . You are given a cost matrix  $C$  of size  $n \times n$ , where  $C(i, j)$  is the rent to be paid for the boat rented at  $i$  and returned at  $j$ ,  $1 \leq i < j \leq n$ . We assume that  $C(i, i) = 0$  and that you cannot go up the river. [Note: Elements below the principle diagonal in the Cost matrix  $C$  is  $\infty$  ]

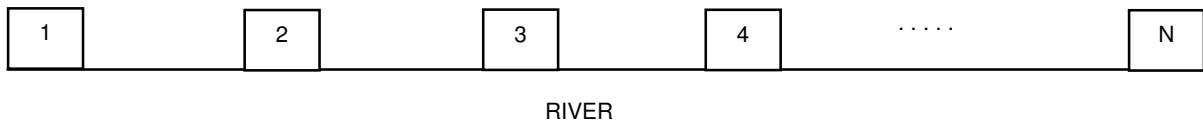


Figure 1

The problem is to find the dynamic programming algorithm that computes minimum cost to be paid to reach shop  $n$ .

- (a) Give the Optimal Substructure equation.

(5)

(b) Also provide an algorithm that uses memoization to solve this problem.

(5)

2. Alisha is a 6 year old girl. She loves Candies. She is given  $n$  boxes( $Candies[1 \dots n]$ ) in a row filled with different number of candies. She can pick anyone  $i$ th box and can eat all the candies in it , along with that she can also eat candies from the boxes having same number (i.e. $Candies[i]$ ) of candies. But there is a trick to it, after she ate all the candies in the boxes having  $Candies[i]$  number of candies she can't touch the boxes have  $Candies[i] - 1$  &  $Candies[i] + 1$  number of candies ever. This goes on till there are no box candies left. Your task is to help Alisha eat maximum number of candies. Write an Algorithm that reports which boxes of candies to be picked in a particular order and also report the maximum number of candies that Alisha can eat. Also give its time complexity. (10)

3. Jasmine wants to fill a bag with balls. The bag can filled to a given capacity limit( $C$ ). There are  $N$  boxes numbered 1 to  $N$  that contain some balls. She noticed that if she picks the ball from box  $i$ , she can't pick the balls from box  $i + 1$ .  
Jasmine wants to fill the bag with maximum balls she can fill. Given the number of boxes and balls in each box and the capacity limit  $C$  of bag, help her to fill the bag with maximum capacity possible. (10)

4. Virat Kohli wants to score  $N$  runs in a match. He can score only runs from this set  $R = \{1, 2, 3, 4, 6\}$ . He can play unlimited balls. He wants to know the number of possible ways he can reach the score  $N$ . Note : Sequence 2 1 and 1 2 should be count as one, For example  $N = 3$  output should be 3. (10)

5. Career Booster Coaching Classes host  $n$  lectures a day. The begin and ending of each lecture is given. Find the minimum number of class rooms needed such that no lecture is delayed. (10)

For example,

$LB = \{2 : 00, 2 : 10, 3 : 00, 3 : 20, 3 : 50, 5 : 00\}$

$LE = \{2 : 30, 3 : 40, 3 : 20, 4 : 30, 4 : 00, 5 : 20\}$

Here,  $i$ th element of  $LB$  is the start timing of the  $i$ th lecture and  $i$ th element of  $LE$  is the end timing of the  $i$ th lecture.

Output : Minimum rooms required is 2

6. In a stock market, there is a product with its infinite stocks. The stock prices are given for  $n$  days, where  $arr[i]$  denotes the price of the stock on the  $i$ th day. There is a rule that a customer can buy at most  $i$  stock on the  $i$ th day. If the customer has an amount of  $k$  rupees initially, find out the maximum number of stocks they can buy? (10)

For example, for 3 days the price of a stock is given as  $\{7, 10, 4\}$ . You can buy stock 1 worth 7 on day 1, 2 stocks worth 10 each on day 2 and 3 stocks worth 4 each on day 3. If  $k = 100$ , you can buy all the stocks (total 6) for 39.