Graph Theory

Tutorial/Homework 7

October 11, 2012

Tutorial 7

1. Use the Konig-Egervary theorem to prove that every bipartite graph has a matching of size at least $e(G)/\Delta(G)$. Use this to prove that every subgraph of $K_{n,n}$ with more than $(k-1)n$ edges has a matching of size at least k.
2. Construct a graph that is neither a complete graph nor an odd cycle, but has a vertex ordering relative to which greedy coloring uses $\Delta(G)+1$ colors.
3. Prove or disprove: For every graph G, $\chi(G) \leq n(G) - \alpha(G) + 1$
4. Prove or disprove: Every k-chromatic graph has some proper k-coloring in which some color class has $\alpha(G)$ colors.

Homework 7

1. A k-factor is a k-regular spanning subgraph of a graph G (so a perfect matching is a 1-factor because all vertices of the graph are in it (so it is a spanning subgraph) and degree of all vertices is 1 (so it is 1-regular)). Prove that every regular graph of positive even degree has a 2-factor. (Hint: Start with an Euler tour)}
2. Two people play a game on a graph G , alternately choosing distinct vertices. Player 1 starts the game by choosing any vertex. Each subsequent choice must be adjacent to the preceding choice (of the other player). Thus together they follow a path. The last player able to move wins. Prove that the second player has a winning strategy if G has a perfect matching, and otherwise, the first player has a winning strategy. (A strategy is a particular choice of moves by a player. A strategy of a player is a winning strategy if it will always result in a win for the player, irrespective of what strategy the other player plays with).

3. Teachers in a school have been asked to nominate, from those students known to them (every teacher may not know all students), three students each as candidates for the school council. At least one student nominated by each teacher is to be in Year 12. No student may be nominated by more than one teacher. The teachers decide to put their heads together to do the nomination. Can you help them in deciding if they can at all meet the nomination requirements, and if so, find such a nomination?}
4. Prove that $\chi(G) = \omega(G)$ if G' (complement of G) is bipartite.