

# Graph Theory

## Tutorial/Homework 3

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# Tutorial 3

1. Show that a graph with  $n$  vertices with vertex connectivity  $k$  has at least  $kn/2$  edges.
2. Prove that every edge cut in a graph must contain one branch from every spanning tree of the graph.
3. Prove that an Eulerian graph cannot have a minimal edge cut with an odd number of edges.

# Homework 3

1. Prove that no graph has a cut vertex of degree 1.
2. Prove that  $\kappa(G) = \kappa'(G)$  when  $G$  is a simple graph with  $\Delta(G) \leq 3$ .
3. Let  $v$  be a cut vertex of a simple connected graph  $G$ . Prove that  $(G' - v)$  is connected.
4. Prove that a simple graph  $G$  is 2-connected if and only if for every triple  $(x, y, z)$  of distinct vertices,  $G$  has an  $(x,z)$  path through  $y$ .
5. Let  $G$  be a connected graph with at least 3 vertices. Form  $G'$  from  $G$  by adding an edge  $(x,y)$  in  $G'$  (if it is not already there in  $G$ ) whenever  $d(x,y) = 2$  in  $G$ . Prove that  $G'$  is 2-connected.