Graph Theory

Tutorial/Homework 2

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Tutorial 2

1. Let T be a tree of order n. Prove that T is isomorphic to a subgraph of the complement of C_{n+2}
2. The girth of a graph G, denoted by g(G), is the length (no. of edges) of the smallest cycle in the graph. Prove that a k-regular graph with girth 4 has at least 2k vertices. What is the class of such graphs (i.e. k-regular with girth 4) with exactly 2k vertices?
3. Prove that for every 3-regular graph, the vertex connectivity is the same as the edge connectivity.

Homework 2

1. Prove that if W is a nontrivial closed walk that does not contain a cycle, then some edge of W occurs twice in succession (once in each direction).
2. Prove that G has a Hamiltonian Path only if for every subset S of V, the number of components of G – S is at most |S| + 1.
3. The girth of a graph G, denoted by g(G), is the length (no. of edges) of the smallest cycle in the graph. Prove that every graph G containing at least one cycle satisfies the relation g(G) ≤ 2diam(G) + 1, where diam(G) is the diameter of G.

4. If G is a graph, then the line graph of G, L(G), is the graph formed as follows: For each edge in G, add a vertex in L(G) and add an edge between two vertices in L(G) if the corresponding edges in G have a common endpoint. Prove that
   (a) If G is Eulerian, then L(G) has a Hamiltonian circuit
   (b) $L(K_{m,n})$ is regular
5. Let v be a vertex in a connected graph G. Prove that there exists a spanning tree T of G such that the distance of every vertex from v is the same in G and in T.