

Graph Theory

Tutorial/Homework 1

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Tutorial 1

1. Prove that every n vertex graph with at least n edges contains a cycle.
2. Consider a k -regular bipartite graph with m and n vertices in the two partite sets. Prove that $m = n$.
3. Suppose a connected graph G is decomposed into two graphs G_1 and G_2 . Prove that G_1 and G_2 must have a common vertex.
4. Let P and Q be two paths of maximum length in a connected graph G . Prove that P and Q must have a common vertex.

Homework 1

1. Isomorphism checking: Pr. 1.16, 1.18. 1.19 from West
2. Check if graphic sequence or not: Pr. 1.3.8 in West
3. Prove or disprove: If G is an Eulerian graph, and there are two edges e and f in G sharing a vertex, then G has an Eulerian circuit in which e and f appear consecutively.

4. Let G be a graph with at least 2 vertices. Prove or disprove the following:
 - Deleting a vertex with degree $\Delta(G)$ cannot increase the average degree of G .
 - Deleting a vertex with degree $\delta(G)$ cannot reduce the average degree of G .
5. Prove that every n -vertex graph with m -edges has at least $m-n+1$ cycles.
6. Let G be a connected simple graph not having P_4 or C_3 as an induced sub-graph. Prove that, G is a complete bipartite graph (where P_n is a path with n vertices and C_n is a cycle with n vertices).