Fault-tolerant Quantum Computation

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Abstract

Quantum Computing is an exciting alternative model to classical computation. In short the theory of quantum computation and quantum information deals with the tasks of computing and information processing that can be accomplished using small systems which follow the fundamental laws of quantum mechanics. The field has received considerable interests after Shor's polynomial time quantum algorithm for factorization of numbers [15], a problem which still has no polynomial algorithm in the classical model and is strongly believed not to belong to the class **P** of *polynomial time solvable* algorithms, and Grover's quantum search algorithm which finds an element in an unsorted array in $O(\sqrt{n})$ time, a task which cannot be done in better than $O(n)$ time in the classical model, where n is the size of the array. One of the most difficult hurdles in quantum computation and information is decoherence of the quantum mechanical systems which makes the results obtained after processing erroneous. Since the quantum mechanical systems are very small it is very difficult to remove decoherence completely out of the system. Hence the task at hand is to develop schemes that guarantee results of satisfiable quality i. e, fault-tolerance. In this direction Shor proved the fault-tolerance the for the circuit model of quantum computation. Circuit model for quantum computation is analogous to the circuit model of classical computation. Our work here is on fault-tolerance of other quantum computing models like that of quantum cellular automata and adiabatic quantum computing both of which have been shown to be equivalent to circuit model of quantum computing [1, 19].

As already mentioned implementation is an important problem in Quantum Computation. Which is why it will be very helpful if one can design a scheme that reduces the complexity of the implementation. So the task at hand right now is to design an architecture which is simple, computationally powerful and robust. Physical complexity can be reduced considerably if there are

symmetries in the design. Quantum cellular automata (QCA) is computation model where the arrangements of the physical qubits and the gate operation are translation invariant. It has been already shown that one-dimensional QCA [20] can simulate any quantum computer. So quantum cellular automata has most of above mentioned qualities except the fact that it is still not known whether we can develop an fault tolerant scheme to implement quantum cellular automata without losing the topological symmetry of the qubits or the translation invariant gate operations.

Consider a one-dimensional chain of N qubits initialized in the state $|00..0\rangle$ which is applied with the transition function

$$T = (\bigotimes_{i=1}^{N-1} \Lambda(Z)_{i,i+1})(\bigotimes_{i=1}^{N} H_i). \tag{0.1}$$

That is in each elementary step we first apply first Hadamard gate to each qubits and then conditional Z to each neighbouring qubits. And in between each transition operation one may apply transition invariant unitary transformation of the form

$$U_A(\alpha) = \bigotimes_{j=1}^{N} exp(i\frac{\alpha}{2}A_j) \tag{0.2}$$

where $j = \{X, Y, Z\}$ and j implies the bit at which the gate is applied. It has been shown by Robert in [19] that one can implement one-dimensional QCA using these operation. In this thesis I implemented this one-dimensional scheme of QCA using surface codes [21]. The physical qubits are arranged in the lattice edges and they satisfy stabilizer operation [10], i. e they remain unchanged on application of some operations

$$S_X(v) = \bigotimes_{e \in v} X_e, \qquad S_Z(f) = \bigotimes_{e \in f} Z_e \tag{0.3}$$

where v and f are the sites and the faces of the lattice and e corresponds to the edges of the faces and sites. This is an important property of surface codes which will be helpful in the fault-tolerance. The logical qubits and logical gate operations are encoded in some form in the surface code and physical gate operations respectively. All the operation that will be done on the qubits of the lattice edges will be translation invariant in the physical layer of the lattice. The scheme that we have developed will have both error detection and correction properties.

The paper entitled "Quantum Computation by Adiabatic Evolution" by Farhi et al. [3] is a seminal work in the area of quantum computation. The

quantum adiabatic thorem [4] can be used to develop algorithm to solve the 3-SAT problem, , 2-SAT on a ring and the Grover Problem as shown in [3]. For the 3-SAT problem the running time for the algorithm is not known but for the 2-SAT on a ring problem the running time is $O(n^3)$, where n is the size of the input. Aharonov et al. in there paper [1] showed that adiabatic quantum computer is polynomially equivalent to the standard quantum circuit model of computing. This equivalence is an important step which will help in designing new quantum algorithms for problems that till now have no known efficient algorithm in the standard quantum circuit model and also help in constructing fault tolerant quantum computers. Since I was working on the fault-tolerance adiabatic quantum computing and error correcting codes are an important technique used in circuit model to make robust systems hence I started studying the paper named "Error Correcting Codes for Adiabatic Quantum Computation" by Jordan et al. [7]. In this paper it was show how addition of a constant energy gap is possible using error correcting codes that protects against $1-$ and $2-$local noises. Stabilizer codes are used to add the constant energy gap to the adiabatic evolution.

Since it has been shown 2-local Hamiltonians can simulate a quantum computer [1] it is very important to investigate few examples of 2-local Hamiltonians under adiabatic evolution. One very good example is the Isisng Model in a transverse magnetic field which solves a very simple problem of finding a n-bit number where all the bits are same. The Ising Hamiltonian is given by the equation

$$H(t) = -t\sum_{i=1}^{N} X_i - (1-t)\sum_{i=1}^{N} Z_i Z_{i+1}, \quad 0 \leq t \leq 1 \tag{0.4}$$

I have studied the Ising model [12] in transverse magnetic field and have worked out the details of the diagonalization of the Ising model with using Jordan-Wigner and Bogoliubov transformations [12] for the cases $n = 2, 3$ and checked the results with the numerical simulations. In addition to the above I numerically plotted the eigen values w.r.t time for the Isisng model for $n = 4, 5, 6$ qubits. I have plotted the eigen values vs time for the encoded Ising model $n = 2, 3$ logical bits case and checked with the eigen values obtained from the analytical calculations done previously. I used 4-qubit encoding from [7] that protects against 1-local noises. For this work I had to learn *Canonical Commutation Relations* (*CCRs*) for Fermions, consequences of the fremionic CCRs, diagonalization of Fermi quadratic Hamiltonian and Jordan-Wigner and Bogoliubov transformations. I followed [12] and [14] for this part of the work.

I worked on JFS [7] encoding for adiabatic quantum computing. For a given Hamiltonian I calculated explicitely the eigen vaules . Worked out the details of how to add a constant energy gap using the stabilizer formalism [10]. I have explicitly calculated the eigen values for the encoded Hamiltonian for JFS codes that protect against 1 - local errors and then extended it for 2-local errors. For calculating the eigen values for the encoded case techniques used were of stabilizer formalism.

I have been working on the error model used in [7] and [5] . The evolution equation is of Lindblad [5] form and stems from the coupling of each qubit in the adiabatic quantum computer to an harmonic oscillator bath. The master equation for the evolution is

$$\frac{d\rho}{dt} = -i[H_S, \rho] - \sum^{a,b} M_{ab}\varepsilon_{ab}(\rho) \tag{0.5}$$

where M_{ab} is a scalar,

$$\varepsilon_{ab}(\rho) = |a\rangle\langle a|\rho + \rho|a\rangle\langle a| - 2|b\rangle\langle a|\rho|a\rangle\langle b| \tag{0.6}$$

is an operator and a, b are the eigen states of $H(S)$. Under the following model is I have numerically simulated the adiabatic passage through the point of smallest gap in the presence of a bath of local harmonic oscillators.

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