Tutorial on

Identity-Based Cryptography

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Short Term Course on Introduction to Cryptography
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Public-Key Cryptography

- Public keys are used for encryption and digital signature verification.
- Private keys are used for decryption and digital signature generation.
- Public keys are accessible to all parties.
- Private keys are to be kept secret.
- How to associate entities with their respective public keys?
- An attacker may present a harmful key as the public key of a victim.
- Before using a public key, one should verify that the key belongs to the claimed party.

Public-Key Certificates

- There is a trusted Certification Authority (CA).
- CA issues public-key certificates to parties.
- A certificate contains a public key, some identifying information of the party to whom the key belongs, a period of validity.
- The certificate is digitally signed by the CA.
- Key compromise and/or malicious activities may lead to revocation of certificates.
- The CA maintains a list of revoked certificates.

Public-Key Certificates: Use

- Alice wants to send an encrypted message to Bob.
- Alice obtains Bob's public-key certificate.
- Alice verifies the signature of the CA on the certificate.
- Alice confirms that Bob's identity is stored in the certificate.
- Alice checks the validity of the certificate.
- Alice ensures that the certificate does not reside in the revocation list maintained by the CA.
- Alice then uses Bob's public key for encryption.

Identity-Based Cryptography: A Viable Substitute

Problems of Public-Key Certificates

- A trusted CA is needed.
- Every certificate validation requires contact with the CA for the verification key and for the revocation list.

Identity-Based Public Keys

- Alice's identity (like e-mail ID) is used as her public key.
- No contact with the CA is necessary to validate public keys.
- A trusted authority is still needed: Private-Key Generator (PKG) or Key-Generation Center (KGC).
- Each party should meet the PKG privately once (registration phase).
- **Limitation:** Revocation of public keys may be difficult.

Identity-Based Key Exchange (IBKE)
Identity-Based Encryption (IBE)
Identity-Based Signatures (IBS)

Historical Remarks

- Shamir (Crypto 1984) introduces the concept of identity-based encryption (IBE) and signature (IBS). He gives a concrete realization of an IBS scheme.
- In early 2000s, bilinear pairing maps are used for concrete realizations of IBE schemes.
- Sakai, Ohgishi and Kasahara (2000) propose an identity-based key-agreement scheme and an IBS scheme.
- Boneh and Franklin (Crypto 2001) propose an IBE scheme. Its security is proved in the random-oracle model.
- Boneh and Boyen (EuroCrypt 2004) propose an IBE scheme whose security is proved without random oracles.
- Joux (ANTS 2004) proposes a pairing-based three-party key-agreement protocol.

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Identity-Based Key Exchange (IBKE)
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Introduction to Bilinear Pairing

- Let G_1, G_2, G_3 be groups of finite order r (usually prime)
- G_1, G_2 are additive, and G_3 multiplicative.
- A **bilinear pairing map** $e : G_1 \times G_2 \rightarrow G_3$ satisfies:
  - $e(P_1 + P_2, Q) = e(P_1, Q)e(P_2, Q)$ and $e(P, Q_1 + Q_2) = e(P, Q_1)e(P, Q_2)$ for all $P, P_1, P_2 \in G_1$ and $Q, Q_1, Q_2 \in G_2$.
  - $e(aP, bQ) = e(P, Q)^{ab}$ for all $P \in G_1$, $Q \in G_2$, and $a, b \in \mathbb{Z}$.
  - e is non-degenerate, that is, $e(P, Q)$ is not the identity of G_3 for some P, Q.
  - e is efficiently computable.
- Example: Weil or reduced Tate pairing over elliptic curves. G_1, G_2 are elliptic-curve groups, G_3 is a subgroup of the multiplicative group of a finite field.
- Special case: $G_1 = G_2 = G$. Example: Distorted Weil or Tate pairing on supersingular curves.

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Diffie–Hellman Problems

- Let G be an additive group of prime order r.
- **Computational Diffie–Hellman Problem (CDHP):** Given $P, aP, bP \in G$, compute abP.
- **Decisional Diffie–Hellman Problem (DDHP):** Given $P, aP, bP, zP \in G$, decide whether $x \equiv ab \pmod{r}$.
- If $e : G \times G \rightarrow G_3$ is a bilinear pairing map, the DDHP is easy: Check whether $e(aP, bP) = e(P, zP)$.
- The CDHP is not known to be aided by e.
- G is called a **gap Diffie–Hellman (GDH)** group.
- **External Diffie–Hellman Assumption (XDH):** Presence of bilinear pairing maps $e : G_1 \times G_2 \rightarrow G_3$ does not make DDHP easy in G_1 or G_2 (different groups).

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Bilinear Diffie–Hellman Problems

- Let $e : G \times G \to G_3$ be a bilinear pairing map.
- **(Computational) Bilinear Diffie–Hellman Problem (BDHP):** Given $P, aP, bP, cP \in G$, compute $e(P,P)^{abc}$.
- **Decisional Bilinear Diffie–Hellman Problem (DBDHP):** Given $P, aP, bP, cP, zP \in G$, decide whether $z \equiv abc \pmod r$ (that is, $e(P,P)^z = e(P,P)^{abc}$).
- **Bilinear Diffie–Hellman Assumption:** The BDHP and DBDHP are computationally infeasible for suitably chosen groups even in the presence of efficiently computable bilinear pairing maps.
- DLP in G should be difficult (as $e(aP, bP)^c = e(P,P)^{abc}$).
- DHP in G should be difficult (as $e(abP, cP) = e(P,P)^{abc}$).

Sakai–Ohgishi–Kasahara (SOK) Key Agreement

Set-up Phase

The PKG/KGC/TA sets up the following parameters.

- Groups G, G_3 of prime order r.
- A bilinear pairing map $e : G \times G \to G_3$.
- A generator P of G.
- A hash function H to map public identities (like e-mail addresses) to elements of G.
- PKG's master secret key $s \in_U \mathbb{Z}_r$.
- PKG's public key $P_{PKG} = sP$.

SOK Key Agreement (Contd)

Registration (Key-Extraction) Phase

- A user Usr meets the PKG securely.
- The PKG hashes the public identity of Usr to generate $P_{Usr} = H(ID_{Usr}) \in G$.
- The PKG delivers $D_{Usr} = sP_{Usr} \in G$ to Usr.

Notes

- Anybody can compute the hashed public identity P_U.
- Computation of D_{Usr} is equivalent to solving DHP in G ($P_{Usr} = uP$, $P_{PKG} = sP$, and $D_{Usr} = usP$). This is assumed to be intractable.
- Alice and Bob securely registers with the PKG to get D_{Alice} and D_{Bob}.
- Anybody can compute P_{Alice} and P_{Bob}.

SOK Key Agreement (Contd)

Key Agreement (Non-interactive)

- Alice computes Bob's hashed identity P_{Bob}.
- Alice computes $S_{Alice} = e(D_{Alice}, P_{Bob})$.
- Bob computes Alice's hashed identity P_{Alice}.
- Bob computes $S_{Bob} = e(P_{Alice}, D_{Bob})$.
- $S_{Alice} = e(D_{Alice}, P_{Bob}) = e(sP_{Alice}, P_{Bob}) = e(P_{Alice}, P_{Bob})^s = e(P_{Alice}, sP_{Bob}) = e(P_{Alice}, D_{Bob})$ is the shared secret.

Security (Based on BDHP)

- Let $P_{Alice} = aP$ and $P_{Bob} = bP$. We have $P_{PKG} = sP$.
- P, aP, bP, sP are known to any attacker.
- The shared secret is $e(P_{Alice}, P_{Bob})^s = e(P,P)^{abs}$.

Joux Three-Party Key Agreement

- Not an identity-based protocol.
- Alice, Bob, and Carol respectively generate $a, b, c \in_U \mathbb{Z}_r$.
- Alice sends aP to Bob and Carol.
- Bob sends bP to Alice and Carol.
- Carol sends cP to Alice and Bob.
- Alice computes $e(bP, cP)^a = e(P, P)^{abc}$.
- Bob computes $e(aP, cP)^b = e(P, P)^{abc}$.
- Carol computes $e(aP, bP)^c = e(P, P)^{abc}$.
- Man-in-the-middle attack possible.

Boneh–Franklin IBE

Set-up Phase

The PKG/KGC/TA sets up the following parameters.

- Groups G, G_3 of prime order r.
- A bilinear pairing map $e : G \times G \to G_3$.
- A generator P of G.
- An encoding function H_1 to map public identities (like e-mail addresses) to elements of G.
- A function $H_2 : G_3 \to \{0, 1\}^n$ (n is the message length).
- PKG's master secret key $s \in_U \mathbb{Z}_r$.
- PKG's public key $P_{PKG} = sP$.

BF IBE (Contd)

Registration (Key-Extraction) Phase

- A user Usr meets the PKG securely.
- The PKG encodes the public identity of Usr to generate $P_{Usr} = H_1(ID_{Usr}) \in G$.
- The PKG delivers $D_{Usr} = sP_{Usr} \in G$ to Usr.

Notes

- Anybody can compute the encoded public identity P_{Usr}.
- Computation of D_{Usr} is equivalent to solving the DHP in G. This is assumed to be intractable.
- Bob (the recipient) securely meets the PKG to get D_{Bob}.
- Anybody can compute P_{Bob}.

BF IBE (Contd)

Encryption

Alice wants to sent $M \in \{0, 1\}^n$ to Bob.

- Alice computes $P_{Bob} = H_1(ID_{Bob})$.
- Alice computes $g = e(P_{Bob}, P_{PKG}) \in G_3$.
- Alice chooses a random $a \in_U \mathbb{Z}_r^*$.
- Alice computes $U = aP$ and $V = M \oplus H_2(g^a)$.
- A ciphertext for M is the pair $(U, V) \in G \times \{0, 1\}^n$.

Note: $H_2(g^a)$ acts as a mask to hide M.

BF IBE (Contd)

Decryption

- Bob recovers M from (U, V) as $M = V \oplus H_2(e(D_{Bob}, U))$.

Correctness

- Let $P_{Bob} = bP$.
- $g^a = e(P_{Bob}, P_{PKG})^a = e(bP, sP)^a = e(P, P)^{abs}$.
- $e(D_{Bob}, U) = e(sP_{Bob}, aP) = e(sbP, aP) = e(P, P)^{abs}$.

Textbook Security

- Malice knows $aP = U$, $bP = P_{Bob}$, and $sP = P_{PKG}$.
- His ability of computing the mask is equivalent to solving an instance of the BDHP.

BF IBE (Contd)

Insecurity against Active Attacks

- Malice wants to get M corresponding to (U, V).
- Malice gets assistance from Bob's decryption box.
- The decryption box decrypts any ciphertext except (U, V).
- The decryption box may refuse to answer if decryption results in the message M.
- Malice queries with $U' = U$ and $V' = W \oplus V$ for some $W \in_U \{0,1\}^n \setminus \{0^n\}$ chosen by Malice.
- $(U', V') \neq (U, V)$ encrypts $M' = M \oplus W$.
- For random W, M' is a random n-bit string.
- The decryption box returns M'.
- Malice computes $M = M' \oplus W$.

IND-CPA (Semantic) Security

The IND-CPA Game

- Malice chooses messages m_0, m_1 of the same bit length.
- Malice sends m_0, m_1 to the victim's encryption oracle $\mathscr{O}$.
- $\mathscr{O}$ chooses a bit $b \in_U \{0, 1\}$, and encrypts m_b.
- The ciphertext c^* of m_b is sent to Malice as the challenge.
- Malice outputs a bit b'. Malice wins if and only if $b' = b$.

Notes

- Encryption must be randomized.
- A random guess of Malice succeeds with probability $1/2$.
- Malice succeeds with probability $1/2 + \varepsilon$ (ε is advantage).
- If ε is less that one over all polynomial expressions in the security parameter, the scheme in IND-CPA secure.

IND-CCA Security

- Malice has access to the victim's decryption oracle $\mathscr{O}$.
- Malice sends indifferent chosen ciphertexts for decryption before the IND-CPA game.
- Malice sends adaptive chosen ciphertexts for decryption after the IND-CPA game.
- Query on c^* cannot be made after the challenge is posed.
- CCA1: Decryption assistance stops after the challenge.
- CCA2: Decryption assistance continues after the challenge.
- The cryptanalysis training before and/or after the challenge is supposed to help Malice in winning.
- CCA2 is the accepted standard model of the adversary.

IND-ID-CPA and IND-ID-CCA Security

- In an IBE scheme, there are registration requests.
- Malice has access to the registration oracle $\mathscr{R}$.
- Malice can make queries to $\mathscr{R}$ before and after the challenge.
- Bob is the targeted victim (c^* is generated by Bob's encryption oracle).
- Malice may never ask $\mathscr{R}$ to reveal Bob's private key.
- Malice may ask $\mathscr{R}$ to reveal Bob's public key (or can compute the public key himself).

Random Oracles

A **random oracle** is a function H from $\{0,1\}^*$ to a finite set D.

- H is deterministic.
- For each input $\alpha \in \{0,1\}^*$, $H(\alpha)$ is a uniformly random element of D.
- H is efficiently computable.

In theory: Random oracles do not exist.

In practice

- H can be treated as a random oracle if its output cannot be distinguished from truly random output by any probabilistic polynomial-time algorithm.
- Cryptographic hash functions are used as random oracles.

Security Proof in the Random-Oracle Model (ROM)

In Real Life

- Malice can compute all hash functions himself.
- Malice can access encryption/decryption/registration oracles.

In ROM Proofs

- Malice communicates only with Ronald.
- Ronald has no access to the victim's/PKG's private keys.
- Ronald has full control over hash computations.
- Malice has to contact Ronald if he wants to hash anything.
- By manipulating hash values, Ronald reliably simulates encryption/decryption/registration queries.
- If the simulation is reliable, Malice unleashes his cryptanalytic prowess to win the game.

Hash Queries

- Ronald maintains a table T of $(\alpha, H(\alpha))$ values.
- Initially, T is empty.
- Whenever some $H(Q)$ needs to be returned, Ronald searches for Q in T.
- If the search is successful, the second stored component is returned.
- If the search is unsuccessful, Ronald chooses a uniformly random $\gamma \in D$, stores (Q, γ) in T, and returns γ.
- The attack runs for polynomial time, so the size of T never grows beyond polynomial. Searching in T is efficient.
- Sometimes additional information is stored in entries of T.

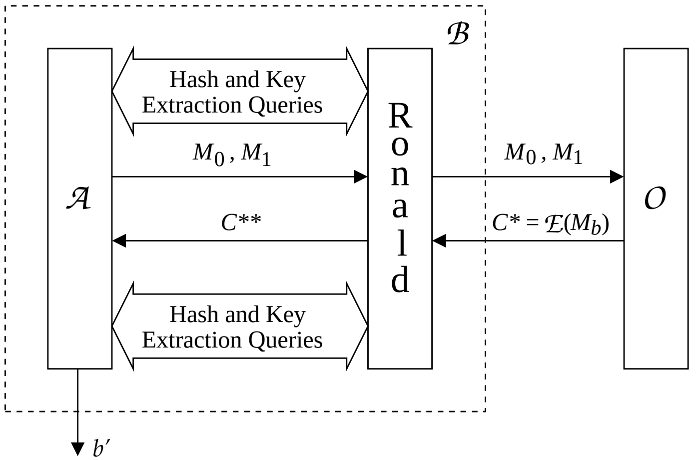
IND-ID-CPA Proof of BF IBE in the ROM

- H_1, H_2 are treated as hash functions (random oracles).
- Step 1: Infeasibility of BDHP in G implies IND-CPA security.
- Step 2: IND-CPA security implies IND-ID-CPA security.
- If there is an IND-ID-CPA adversary $\mathscr{A}$ for BF IBE, then there is an IND-CPA adversary $\mathscr{B}$ for BF IBE.
- If there is an IND-CPA adversary $\mathscr{B}$ for BF IBE, then Ronald can reliably solve the BDHP in G.
- Let the advantage of $\mathscr{A}$ be ε.
- Let the number of H_1 and H_2 queries be q_{H_1} and q_{H_2}.
- Then, the advantage of $\mathscr{B}$ is $\frac{\varepsilon}{e(1+q_{H_1})}$, and the advantage of Ronald in solving the BDHP is $\frac{2\varepsilon}{e(1+q_{H_1})q_{H_2}}$.

IND-CPA Security Implies IND-ID-CPA Security

- Let $\mathscr{A}$ be a PPT IND-ID-CPA adversary.
- Ronald interacts with $\mathscr{A}$ and $\mathscr{O}$.
- System parameters $G, G_3, r, e, P, P_{PKG}, n, H_2$ are public.
- The master secret s is fixed, but not known to $\mathscr{A}$, Ronald, or $\mathscr{O}$.
- Bob is the targeted victim decided by $\mathscr{A}$.
- A registration query to get D_{Bob} cannot be made by $\mathscr{A}$.
- A query to get $P_{Bob} = H_1(ID_{Bob})$ is allowed. $\mathscr{A}$ cannot know P_{Bob} without making this query.
- H_1 is a random oracle to $\mathscr{A}$.
- The encryption oracle $\mathscr{O}$ uses actual hash values. $P_{Bob}^{(\mathscr{O})}$ and $D_{Bob}^{(\mathscr{O})}$ are the actual (not simulated) keys of Bob. Both Ronald and $\mathscr{O}$ knows how to compute $P_{Usr}^{(\mathscr{O})}$ for any Usr.

The Reduction Mechanism



Handling H_1 Queries

Key Extraction: $P_{Usr} = H_1(ID_{Usr})$, $D_{Usr} = sP_{Usr}$.
Encryption: $U = aP$, $g = e(P_{Bob}, P_{PKG})$, $V = M \oplus H_2(g^a)$.
Decryption: $M = V \oplus H_2(e(D_{Bob}, U))$.

- H_1 hashes public ID's to public keys.
- Public keys are needed for key extraction and encryption.
- Ronald does not know s. Let $P_{Usr} = tP$ (where $Usr \neq Bob$). Then, $D_{Usr} = sP_{Usr} = stP = t(sP) = tP_{PKG}$.
- If $Usr = Bob$, D_{Bob} is not needed. Let $P_{Bob} = tP_{Bob}^{(\mathscr{O})}$, and $C^* = (U^*, V^*)$. Then, $e(D_{Bob}^{(\mathscr{O})}, U^*) = e(t^{-1}D_{Bob}, U^*) = e(D_{Bob}, t^{-1}U^*)$. So if $C^* = (U^*, V^*)$ is an actual encryption of M_b done by $\mathscr{O}$, then $C^{**} = (t^{-1}U^*, V^*)$ is an encryption of M_b simulated by Ronald.
- When a query $H_1(ID_{Usr})$ comes, Ronald need not know whether Usr is the targeted victim.

Handling H_1 Queries (Contd)

- Ronald maintains an H_1-table of $(ID_{Usr}, P_{Usr}, t, c)$ entries.
- Suppose that a query $H_1(ID_{Usr})$ comes.
- If ID_{Usr} resides in the H_1-table, the corresponding P_{Usr} is returned.
- Otherwise, Ronald tosses a coin to get c such that $\Pr[c = 0] = \delta \approx 1$.
- If $c = 0$, Ronald assumes $ID \neq Bob$. He chooses random $t \in \mathbb{Z}_r^*$, computes $P_{Usr} = tP$, stores $(ID_{Usr}, P_{Usr}, t, 0)$ in his H_1-table, and returns P_{Usr}.
- If $c = 1$, Ronald assumes $ID = Bob$. He chooses random $t \in \mathbb{Z}_r^*$, computes $P_{Usr} = tP_{Usr}^{(\mathscr{O})}$, stores $(ID_{Usr}, P_{Usr}, t, 1)$ in his H_1-table, and returns P_{Usr}.

Identity-Based Key Exchange (IBKE) Security Models
Identity-Based Encryption (IBE) Security Proof
Identity-Based Signatures (IBS) Boneh–Boyen Encryption

Handling Key-Extraction Queries

- $\mathscr{A}$ asks Ronald to supply the private key D_{Usr} of Usr.
- Ronald searches for ID_{Usr} in his H_1-table.
- If the search fails, Ronald initiates an internal query for computing $H_1(ID_{Usr})$ (he may force $c = 0$ in this query).
- If the H_1-table contains an entry $(ID_{Usr}, P_{Usr}, t, c)$ with $c = 1$, Ronald aborts.
- Finally, suppose that the H_1-table contains an entry $(ID_{Usr}, P_{Usr}, t, c)$ with $c = 0$. Ronald computes and returns $D_{Usr} = tP_{PKG}$.
- Ronald successfully handles a key-extraction query with probability δ.

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Identity-Based Key Exchange (IBKE) Security Models
Identity-Based Encryption (IBE) Security Proof
Identity-Based Signatures (IBS) Boneh–Boyen Encryption

Handling the IND-CPA Game

- $\mathscr{A}$ sends the ID of a targeted victim Bob, and two messages M_0, M_1 of length n, to Ronald.
- Ronald searches for ID_{Bob} in his H_1-table.
- If the search fails, Ronald initiates an internal query for computing $H_1(ID_{Bob})$ (he may force $c = 1$ in this query).
- If the H_1-table contains an entry $(ID_{Bob}, P_{Bob}, t, c)$ with $c = 0$, Ronald aborts.
- Finally, suppose that the H_1-table contains an entry $(ID_{Bob}, P_{Bob}, t, c)$ with $c = 1$.
  - Ronald forwards ID_{Bob}, M_0, M_1 to $\mathscr{O}$.
  - $\mathscr{O}$ chooses $b \in_U \{0, 1\}$, and returns an actual (not simulated) encryption $C^* = (U^*, V^*)$ of M_b using Bob's public key.
  - Ronald forwards $C^{**} = (t^{-1}U^*, V^*)$ to $\mathscr{A}$.
- Ronald successfully participates in the IND-CPA game with probability $1 - \delta$.

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Identity-Based Key Exchange (IBKE) Security Models
Identity-Based Encryption (IBE) Security Proof
Identity-Based Signatures (IBS) Boneh–Boyen Encryption

Advantage of $\mathscr{B}$ (Ronald)

- Let $\mathscr{A}$ have a non-negligible advantage ε.
- If Ronald does not abort, his simulation is perfect. In this case, he has the same advantage ε.
- Let q_{H_1} be the number of H_1-queries made.
- Ronald does not abort with probability $\delta^{q_{H_1}}(1 - \delta)$.
- This probability is maximized for $\delta = \frac{q_{H_1}}{q_{H_1}+1}$.
- The maximum is approximately $\frac{1}{e(q_{H_1}+1)}$.
- Ronald's advantage in winning the IND-CPA game is therefore $\frac{\varepsilon}{e(q_{H_1}+1)}$.
- If Bob is known to be the targeted victim at the beginning, all H_1 queries can be answered appropriately, and Ronald never aborts (selective-ID or IND-sID security).

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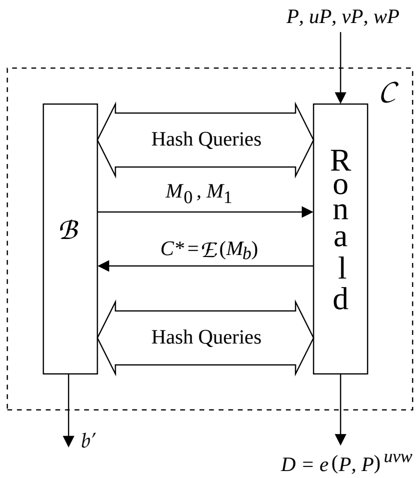
BDH Assumption Implies IND-CPA Security

- Let $\mathscr{B}$ be a PPT IND-CPA adversary.
- Then, there exists a PPT algorithm $\mathscr{C}$ to solve the bilinear Diffie–Hellman problem.
- $\mathscr{C}$ takes P, uP, vP, wP as inputs, and returns $D = e(P, P)^{uvw}$.
- $\mathscr{C}$ consists of $\mathscr{B}$ and Ronald (no external oracle $\mathscr{O}$ now).
- All interactions are between $\mathscr{B}$ and Ronald.
- System parameters $G, G_3, r, e, P, P_{PKG}, n, H_1$ are public.
- Bob is the targeted victim from the beginning.
- $\mathscr{C}$ sets and publicizes $P_{PKG} = uP$ and $P_{Bob} = vP$.
- The master secret is therefore u.
- Bob's private key $D_{Bob} = uP_{Bob} = uvP$ is unknown.
- H_2 is now a random oracle to $\mathscr{B}$.

The Reduction Mechanism



Handling H_2 Queries

- Ronald maintains an H_2-table of (Q, W) pairs $(W = H_2(Q))$.
- Suppose that a query $H_2(Q)$ comes.
- If some (Q, W) is found in the H_2-table, W is returned as $H_2(Q)$.
- Otherwise, Ronald chooses $W \in_U \{0, 1\}^n$, stores (Q, W) in his H_2-table, and returns W.
- Hash queries are not *manipulated* here.

Handling the IND-CPA Game

- $\mathscr{B}$ sends two messages M_0, M_1 of length n to Ronald.
- Ronald takes $U^* = wP$ and $V^* \in_U \{0, 1\}^n$, and sends the challenge ciphertext $C^* = (U^*, V^*)$ as a purported encryption of M_b (for some $b \in_U \{0, 1\}$).
- $P_{PKG} = uP$, $P_{Bob} = vP$, and $U^* = wP$, so the mask before hashing is $e(P_{Bob}, P_{PKG})^w = e(vP, uP)^w = e(P, P)^{uvw} = D$.
- If $H_2(D) = V^* \oplus M_b$, then C^* is a valid ciphertext for M_b.
- $\mathscr{B}$ makes an H_2-query on D in the post-challenge phase with very high probability, so D ends up in Ronald's H_2-table.
- Ronald cannot identify which is the correct D (difficulty of the decisional BDH problem).
- Ronald chooses a random (Q, W) entry from his H_2-table, and returns W as $D = e(P, P)^{uvw}$.

Advantage of $\mathscr{C}$ (Ronald)

- Let the advantage of $\mathscr{B}$ be ε' for winning the IND-CPA game.
- The actual D is queried (to the random oracle H_2) with probability $\geqslant 2\varepsilon'$.
- Let q_{H_2} denote the number of H_2 queries.
- Since an entry of the H_2-table is chosen at random, the advantage of $\mathscr{C}$ is $\geqslant 2\varepsilon'/q_{H_2}$.

From IND-CPA to IND-CCA Security

- The Fujisaki–Okamoto transform converts an IND-CPA secure encryption scheme to an IND-CCA secure scheme.
- Two additional hash functions $H_3 : \{0,1\}^n \times \{0,1\}^n \to \mathbb{Z}_r^*$ and $H_4 : \{0,1\}^n \to \{0,1\}^n$ are used.
- **Encryption** of $M \in \{0,1\}^n$ is (U, V, W).
  - Compute $P_{Bob} = H_1(ID_{Bob}) \in G$.
  - Choose $\sigma \in_U \{0,1\}^n$, and compute $a = H_3(\sigma, M)$.
  - Compute $g = e(P_{Bob}, P_{PKG})$.
  - $U = aP$, $V = \sigma \oplus H_2(g^a)$, and $W = M \oplus H_4(\sigma)$.
- **Decryption** of (U, V, W):
  - Recover $\sigma = V \oplus H_2(e(D_{Bob}, U))$.
  - Recover $M = W \oplus H_4(\sigma)$.
  - Set $a = H_3(\sigma, M)$. If $U \neq aP$, return *failure*.
  - Return M.

From IND-CCA to IND-ID-CCA Security

- A reduction similar to the IND-CPA to IND-ID-CPA security works.
- Now, Ronald has to handle decryption queries like (ID_{Usr}, U, V, W).
- Ronald locates $(ID_{Usr}, P_{Usr}, t, c)$ in his H_1-table. If such an entry does not exist, it is created.
- If $c = 0$, Ronald computes the private key $D_{Usr} = tP_{PKG}$, and carries out the decryption himself.
- If $c = 1$, Ronald forwards the query (ID_{Usr}, tU, V, W) to the external decryption oracle $\mathscr{O}$, and relays the response of $\mathscr{O}$ back to $\mathscr{A}$.
- Each decryption query is perfectly answered by Ronald.

Boneh–Boyen IBE

Setup Phase

- G (additive) and G_3 (multiplicative) are groups of prime order r. P is a generator of G.
- $e : G \times G \to G_3$ is a bilinear pairing map.
- Master secret key of PKG: two integers $s_1, s_2 \in \mathbb{Z}_r^*$.
- Public key of PKG: the elements $Y_1 = s_1P$ and $Y_2 = s_2P$ of G.

Registration Phase

- Let $P_{Bob} \in \mathbb{Z}_r^*$ be the hashed public identity of Bob.
- The PKG generates $t \in_U \mathbb{Z}_r^*$, and computes $D = (P_{Bob} + s_1 + s_2t)^{-1}P \in G$.
- Bob's private key is (t, D).
- **Note:** Registration phase is randomized.

Boneh–Boyen IBE (Contd)

Encryption of $M \in G$

- Alice generates $k \in_U \mathbb{Z}_r^*$.
- Alice computes $U = kP_{Bob}P + kY_1 \in G$, $V = kY_2 \in G$, and $W = M \times e(P, P)^k \in G_3$.
- The ciphertext is the triple (U, V, W).

Decryption of (U, V, W)

- $U + tV = k(P_{Bob} + s_1 + s_2 t)P$.
- $e(U + tV, D) = e(k(P_{Bob} + s_1 + s_2 t)P, (P_{Bob} + s_1 + s_2 t)^{-1} P) = e(P, P)^k$.
- $M = W \times e(U + tV, D)^{-1}$.

Boneh–Boyen IBE: Security

- **q-BDHI Problem:** Given $P, aP, a^2P, a^3P, \ldots, a^qP \in G$, compute $e(P, P)^{a^{-1} \pmod r}$ (I in BDHI is Inversion).
- **Decisional q-BDHI Problem:** Given $P, aP, a^2P, a^3P, \ldots, a^qP \in G$ and $T \in G_3$, decide whether $T = e(P, P)^{a^{-1} \pmod r}$.
- **q-BDHI assumption:** These problems are infeasible.
- Boneh–Boyen encryption is IND-sID-CPA secure for a pre-selected victim (Bob) if the decisional q-BDHI assumption holds, where q is the maximum number of key-extraction queries allowed.
- The proof does not require random oracles.
- Using a transform proposed by Canetti et al., the scheme can be made IND-sID-CCA secure.

Shamir's IBS

Setup Phase

- PKG generates an RSA modulus $n = pq$, and computes $\phi(n) = (p-1)(q-1)$.
- PKG chooses $e \geqslant 3$ such that $\gcd(e, \phi(n)) = 1$, and computes $d \equiv e^{-1} \pmod{\phi(n)}$.
- PKG fixes a hash function $H : \{0,1\}^* \to \mathbb{Z}_n$.
- PKG publishes n, e, H.
- $p, q, \phi(n), d$ are kept secret.

Registration Phase

- PKG computes Bob's hashed public identity $P_{Bob} = H(ID_{Bob})$.
- Bob's private key: $D_{Bob} \equiv P_{Bob}^d \pmod n$.

Shamir's IBS (Contd)

Signature Generation

- Bob chooses $x \in_U \mathbb{Z}_n$.
- Bob computes $s \equiv x^e \pmod n$ and $t \equiv D_{Bob} \times x^{H(s,M)} \pmod n$.
- Bob's signature on M is the pair (s, t).

Signature Verification

- $t^e \equiv P_{Bob} \times (x^e)^{H(s,M)} \equiv P_{Bob} \times s^{H(s,M)} \pmod n$.

Security

- A forger can generate $x, s, H(s, M)$.
- Generating the correct t is equivalent to knowing D_{Bob}.
- Getting D_{Bob} from P_{Bob} is the RSA problem.

Sakai–Ohgishi–Kasahara (SOK) IBS

Setup Phase

- G (additive) and G_3 (multiplicative) are groups of prime order r. P is a generator of G.
- $e : G \times G \to G_3$ is a bilinear pairing map.
- Master secret key of PKG: $s \in_U \mathbb{Z}_r^*$.
- Public key of PKG: $P_{PKG} = sP \in G$.
- $H : \{0,1\}^* \to G$ is a public hash function.

Registration Phase

- Bob's public key: $P_{Bob} = H(ID_{Bob}) \in G$.
- Bob's private key: $D_{Bob} = sP_{Bob} \in G$.

SOK IBS (Contd)

Signature Generation

- Bob chooses $d \in_U \mathbb{Z}_r$, and computes $U = dP \in G$.
- Bob also computes $h = H(P_{Bob}, M, U) \in G$ and $V = D_{Bob} + dh \in G$.
- Bob's signature on M is (U, V).

Signature Verification

- $$\begin{aligned} e(P,V) &= e(P, D_{Bob} + dh) \\ &= e(P, sP_{Bob} + dh) \\ &= e(P, sP_{Bob})e(P, dh) \\ &= e(sP, P_{Bob})e(dP, h) \\ &= e(P_{PKG}, P_{Bob})e(U, H(P_{Bob}, M, U)). \end{aligned}$$

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Thank You

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