CS21002 Switching Circuits and Logic Design, Spring 2015–2016

Class Test 1

12–February–2016 | 6:00–7:00pm | Maximum marks: 20

Roll no: ____________ **Name:** ______________________________

[*Write your answers in the question paper itself. Be brief and precise. Answer all questions.*]

1. Suppose that you want to encode the four-bit primes $2,3,5,7,11,13$ (there are six of them). You need a one-error-correcting encoding. If you follow the Hamming encoding technique, you have a seven-bit one-error-correcting encoding of the binary-coded primes. Prove or disprove: It is possible to design a six-bit one-error-correcting encoding of the six primes. You can assign any six-bit codewords to the six primes. There is no necessity to append two bits to the four-bit binary-coded values. **(5)**

Solution Yes, it is possible. Here is an encoding scheme. It is easy to verify that the minimum distance of this encoding is three. So this code is one-error-correcting.

Prime	Encoding
2	000000
3	000111
5	011100
7	110001
11	101010
13	111111

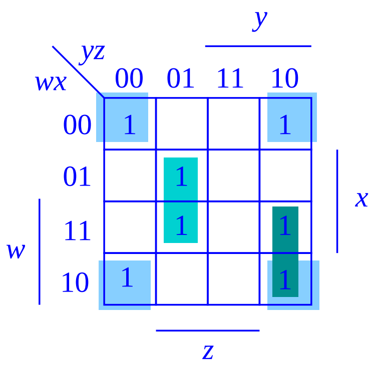
Instead of making a heuristic search, one can proceed systematically. Since there are only six elements to encode, a three-bit encoding suffices. We start with such an arbitrary encoding—call it xyz. We then add three parity-check bits p,q,r satisfying $x \oplus y \oplus p = y \oplus z \oplus q = z \oplus x \oplus r = 0$. An example follows.

Prime	xyz	pqr
2	000	000
3	001	011
5	010	110
7	011	101
11	100	101
13	101	110

Now, take the initial three-bit codewords for two primes. If they differ in one or two bit position(s), then two parity-check bits will be different, so the distance between the two six-bit codewords will be three or four. If they differ in three bit positions, then all the three parity check bits will be the same. Nevertheless, the distance between the two six-bit codewords will be three.

2. Find a minimum sum-of-products expression of the function $f(w,x,y,z) = \prod M(1,3,4,6,7,9,11,12,15)$ using Karnaugh maps. **(5)**

Solution We have $f' = \sum m(1,3,4,6,7,9,11,12,15)$, that is, $f = \sum m(0,2,5,8,10,13,14)$. The Karnaugh map for this function is given below.



From the map, we get $f(w,x,y,z) = x'z' + xy'z + wyz'$.

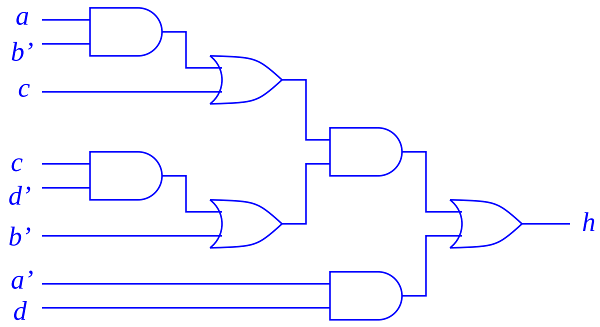
3. You are given three-input logic gates each realizing the function $g(x,y,z) = (x+y')(x'+z)$. Prove that using these gates only, you can realize any Boolean function. **(5)**

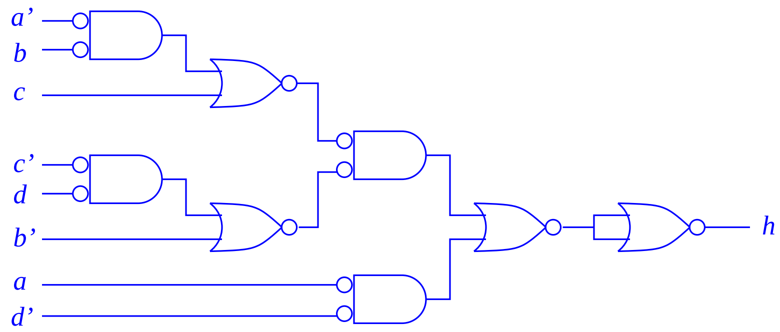
Solution We have $g(x,0,0) = x'$, and $g(x',y,1) = g(g(x,0,0),y,1) = x'+y' = (xy)'$, that is, we can realize a NAND gate using the given gates only.

One may also show the explicit implementation of OR and AND as $g(x,y',1) = g(x,g(y,0,0),1) = x+y$ and $g(g(g(x,0,0),y,1),0,0) = xy$.

4. Make a multi-level realization of $h(a,b,c,d) = (a + b'c)(b' + cd') + a'd$ using two-input NOR gates only. Assume that the four input variables and their complements are available to you. **(5)**

Solution First, draw an AND-OR circuit, and then convert it to a NOR circuit.





For leftover answers and rough work