CS30053 Foundations of Computing, Autumn 2005

Mid-semester examination

Total marks: 60 September 19, 2005 **Duration:** 2 hours

[*Answer* any five *questions*]

1. Consider the language

$$L_1 = \{\alpha \in \{a, b, c\}^* \mid \text{ the last two symbols of } \alpha \text{ are different}\}.$$

(a) Write a regular expression which generates the language L_1. **(6)**

(b) Design an NFA with five states to recognize L_1. **(6)**

2. Let L be a regular language and n a pumping lemma constant for L. Clearly, any integer $\geqslant n$ can also be used as a pumping lemma constant for L. The smallest positive integer which is a pumping lemma constant for L is called the *minimum pumping lemma constant* for L. Determine the minimum pumping lemma constants for the languages over $\{a, b\}^*$ defined by the following regular expressions.

(a) $(ab) \cup (ba)$. **(6)**

(b) $((ab) \cup (ba))^*$. **(6)**

3. Let $\alpha = a_1 a_2 \ldots a_n$ be a string of length n. A string β is called a prefix of α if $\beta = a_1 a_2 \ldots a_i$ for some $i \in \{0, 1, 2, \ldots, n\}$ (the case $i = 0$ corresponds to $\beta = \epsilon$). Consider the language

$$L_3 = \{\alpha \in \{a, b\}^* \mid \text{ no prefix of } \alpha \text{ contains less } a\text{'s than } b\text{'s}\}.$$

(a) Design a PDA to recognize L_3. **(6)**

(b) Design a CFG G with $\mathcal{L}(G) = L_3$. **(6)**

4. Consider the language

$$L_4 = \{\alpha \in \{a, b\}^* \mid |\alpha| = n^2 \text{ for some integer } n \geqslant 0\},$$

where $|\alpha|$ denotes the length of α.

(a) Prove that L_4 is not context-free. **(6)**

(b) Prove that the complement $\overline{L_4} = \{a, b\}^* \setminus L_4$ is also not context-free. **(6)**

5. Let G be the context-free grammar $G = (\{S\}, \{a, b\}, S, R)$ with R consisting of the following rules:

$$S \to \epsilon \mid aS \mid aSb\,.$$

(a) Prove that G is ambiguous. **(6)**

(b) Provide an unambiguous grammar for $\mathcal{L}(G)$. **(6)**

6. (a) Let L and L' be context-free languages. Demonstrate by an example that the language $L \setminus L'$ is *not* necessarily context-free. **(6)**

(b) Prove that if L is a context-free language and R a regular language, then $L \setminus R$ is context-free. **(6)**

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