CS30053 Foundations of Computing, Autumn 2005

End-semester examination

Total marks: 100 | November 23, 2005 | **Duration:** 3 hours

[Answer all questions]

1. Prove that the following languages are not regular. **(10×2)**

(a) $L_{11} = \{a^m b^n \mid m, n > 0 \text{ and } \gcd(m, n) = 1\} \subseteq \{a, b\}^*$.

(b) $L_{12} = \{a^m b^n \mid m, n > 0 \text{ and } \gcd(m, n) > 1\} \subseteq \{a, b\}^*$.

2. Let $\Sigma = \{a, b, c, d\}$. Consider the language

$$L_2 = \{a^i b^j c^k d^l \mid i, j, k, l \geqslant 0 \text{ and } i + j = k + l\} \subseteq \Sigma^*.$$

(a) Design a context-free grammar G with $\mathcal{L}(G) = L_2$. **(10)**

(b) Design a PDA to recognize L_2. **(10)**

3. Prove that the following languages are decidable. Provide only high-level descriptions of deciders. **(5×5)**

(a) $\mathrm{A}_{\mathrm{DFA},n} = \{\langle D, n\rangle \mid D \text{ is a DFA that accepts some string of length } n\}$.

(b) $\mathrm{SUBSET}_{\mathrm{DFA}} = \{\langle D_1, D_2\rangle \mid D_1, D_2 \text{ are DFA with } \mathcal{L}(D_1) \subseteq \mathcal{L}(D_2)\}$.
(Hint: Look at $\mathcal{L}(D_1) \setminus \mathcal{L}(D_2)$.)

(c) $\mathrm{FINITE}_{\mathrm{PDA}} = \{\langle P\rangle \mid P \text{ is a PDA with } \mathcal{L}(P) \text{ finite}\}$.
(Hint: Let n be a pumping lemma constant for $\mathcal{L}(P)$. First prove that $\mathcal{L}(P)$ is infinite if and only if $\mathcal{L}(P)$ contains a string of length between n and $2n - 1$.)

(d) $\mathrm{MOVE}_{\mathrm{TM},\alpha} = \{\langle M, \alpha\rangle \mid M \text{ is a TM that makes at least ten moves on input } \alpha\}$.

(e) $\mathrm{MOVE}_{\mathrm{TM},n} = \{\langle M, n\rangle \mid M \text{ is a TM that makes at least } n \text{ moves on some input}\}$.
(Hint: First argue that it suffices to restrict attention only to input strings of length $\leqslant n$.)

4. Consider the language

$$L_4 = \{\langle M\rangle \mid M \text{ is a TM which halts on the input } 01011\}.$$

Prove the following assertions:

(a) L_4 is Turing-recognizable. **(5)**

(b) L_4 is not Turing-decidable. **(5)**

(c) $\overline{L_4}$ is not Turing-recognizable. **(5)**

5. Consider the language

$$L_5 = \{\langle M\rangle \mid M \text{ is a TM which halts on every input}\}.$$

(a) Use a reduction from $\overline{L_4}$ to L_5 to prove that L_5 is not Turing-recognizable.
(Hint: Suppose that $\langle M\rangle$ maps to $\langle M'\rangle$ under the reduction. Let M' simulate M on input 01011 for n steps, where n is the length of the input string for M'.) **(10)**

(b) Use a reduction from $\overline{L_4}$ to $\overline{L_5}$ to prove that $\overline{L_5}$ is also not Turing-recognizable. **(10)**

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