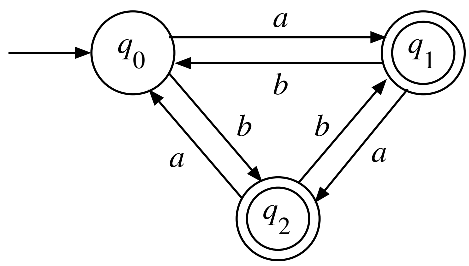
CS30053 Foundations of Computing, Autumn 2005

Class test 1 : Solutions

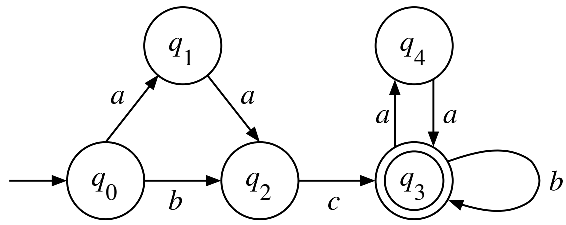
Answer all questions in the respective spaces provided.
Use extra sheets for rough work. Any such extra sheet will not be corrected.

1. Design a finite automaton (deterministic or nondeterministic) to recognize the language **(5)**

$$\{\alpha \in \{a, b\}^* \mid \text{the number of } a\text{'s in } \alpha \text{ minus the number of } b\text{'s in } \alpha \text{ is not a multiple of } 3\} .$$



2. Design a finite automaton (deterministic or nondeterministic) to recognize the set of strings over the alphabet $\{a, b, c\}$, generated by the regular expression $(aa \cup b)c(aa \cup b)^*$. **(5)**



3. Consider the language

$$L = \{a^i b^j c^k \in \{a, b, c\}^* \mid i, j, k \geqslant 0 \text{ and } i + j = k\}.$$

(a) Prove that L is not regular. **(5)**

Assume that L is a regular language. Let n be a pumping lemma constant for L. Then the string $\alpha = a^n c^n$ belongs to L and has length $\geqslant n$. So by the pumping lemma we have a decomposition $\alpha = \alpha_1\alpha_2\alpha_3$. Since $|\alpha_1\alpha_2| \leqslant n$, α_2 consists only of a's. Moreover, $\alpha_2 \neq \epsilon$. Finally, $\alpha_1\alpha_3 \in L$, but $\alpha_1\alpha_3$ lacks the requisite counts of a's (and b's) to deserve inclusion in L, a contradiction.

(b) Design a context-free grammar G with $\mathcal{L}(G) = L$. **(5)**

The CFG $G = (\{S, T\}, \{a, b, c\}, R, S)$ has $\mathcal{L}(G) = L$, where the rules of R are:

$$\begin{array}{lcl} S & \rightarrow & aSc \mid T \\ T & \rightarrow & bTc \mid \epsilon \end{array}$$