CS30053 Foundations of Computing, Spring 2004

Practice exercises : Set 5

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1. Give English-language descriptions of the languages generated by the following phase-structure grammars. In each case, the alphabet of terminal symbols is mentioned at the right end.

(a) $S \to AB$, $A \to \lambda \mid aA$, $B \to b \mid bB$. $\left[\Sigma = \{a, b\}\right]$

(b) $S \to \lambda \mid aT \mid bT \mid cT$, $T \to \lambda \mid aS \mid bS \mid cS$. $\left[\Sigma = \{a, b, c\}\right]$

(c) $S \to \lambda \mid aT \mid bT \mid cT$, $T \to aS \mid bS \mid cS$. $\left[\Sigma = \{a, b, c\}\right]$

(d) $S \to aT \mid bT \mid cT$, $T \to aU \mid bU \mid cU$, $U \to \lambda \mid aS \mid bS \mid cS$. $\left[\Sigma = \{a, b, c\}\right]$

(e) $S \to 0T1$, $T \to \lambda \mid 0T \mid 1T$. $\left[\Sigma = \{0, 1\}\right]$

* **(f)** $S \to \lambda \mid 0S1 \mid 1S0 \mid SS$. $\left[\Sigma = \{0, 1\}\right]$

2. Let G be the context-free grammar defined by the productions $S \to \lambda \mid 0 \mid 1 \mid 0S1 \mid 1S0$. Call $L := \mathcal{L}(G)$. Determine which of the following strings α can be generated by this grammar. If $\alpha \in L$, describe a derivation for α and sketch the corresponding parse tree.

(a) 010101. **(b)** 0101010. **(c)** 11001100. **(d)** 1100110011. **(e)** 10011000110.

3. Write phase-structure grammars for each of the following languages:

(a) The set of all strings over $\{a, b\}$ that start with aa and end with bb.

(b) The set of all strings over $\{a, b\}$ that start with ab and end with ba.

(c) The set of all terminating decimal expansions of real numbers. (The decimal point and the sign (plus or minus) are optional. That is, in addition to $+3.1415926535$, -0.7182818284, your grammar should also generate $+3.1415926535$, 3, 3., $-.1415926535$, etc.)

(d) The set of all arithmetic expressions involving the operators $+$, $-$, $\times$ and $/$ and positive decimal integers. You should also support balanced parentheses. Examples of strings to be generated by your grammar include $12{+}34{\times}56$, $(12{+}34){\times}56$, $210{-}((12{+}34){\times}((56{-}78)/(78{-}34)){-}98)$.

(e) $\{a^{2k+3}b^{3k+2} \mid k \in \mathbb{N}\}$.

(f) The set of all strings over $\{(,)\}$ having balanced parentheses.

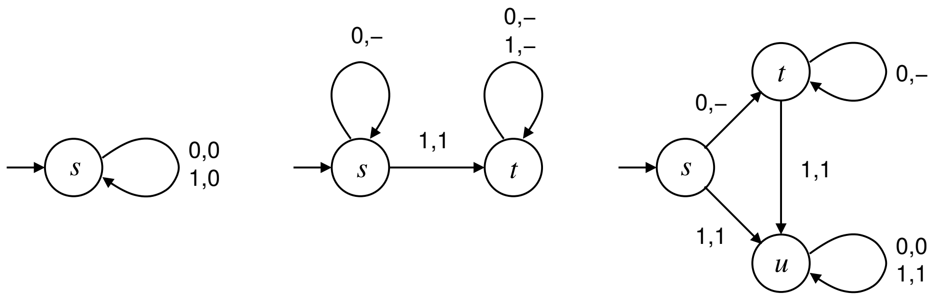
(g) The set of all strings over $\{(,), [,]\}$ having balanced parentheses and brackets.

* **(h)** The set of strings over $\{0, 1\}$ having equal numbers of 0's and 1's.

** **(i)** $\{a^k b^{k^2} \mid k \in \mathbb{N}\}$.

** **(j)** The set of all strings of the form $\alpha\alpha$ for any $\alpha \in \{a, b\}^*$.

4. Write the state transition and output tables for the following finite state machines with outputs. Also provide English-language descriptions of the input/output behaviors of the machines.



(a) (b) (c)

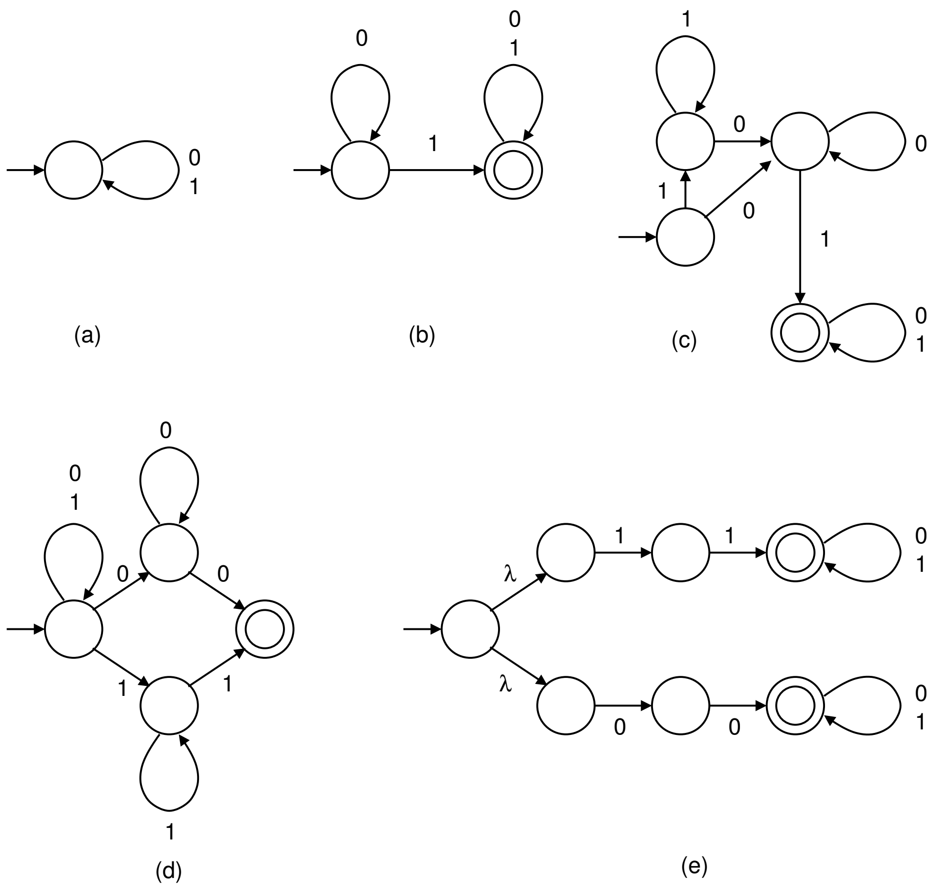
5. Design finite state machines with output for the following input/output specifications:

(a) **Input**: A string α over $\{0, 1\}$.
Output: The bit-wise complement $\overline{\alpha}$ of α.

(b) **Input**: A string α over $\{a, b, c\}$.
Output: The first n symbols of α, where $n = \min(5, |\alpha|)$.

(c) A vending machine that accepts 1, 2 and 5 Rupee coins and sells choco wafers (5 Rs a piece), lolly pops (2 Rs a piece) and chewing gums (4 Rs per packet). The machine should have three buttons, one for each commodity it sells. The exact change must be returned after the user buys an item.

6. Write English-language descriptions of the languages accepted by the following finite automata:



7. Design DFA to accept the following languages:

(a) The set of strings over $\{0, 1\}$ that start with 00 and end with 11.

(b) The set of strings over $\{0, 1\}$ that start with 01 and end with 10.

(c) The set of strings over $\{0, 1\}$ that contain both 01 and 10.

(d) The set of strings over $\{0, 1\}$ that have odd number of 0's.

(e) The set of strings over $\{0, 1\}$ that have odd number of 0's and even numbers of 1's.

(f) $\{0, 1\}^* \setminus \{11, 111\}$.

(g) The binary representations of all non-negative integers that, when divided by 5, give remainders 2 or 3.

8. Design NFA (or λ-NFA) to accept the following languages:

(a) The set of strings over $\{0, 1\}$ that have at least one 0 in the last five positions.

(b) The set of strings over the (lower-case) Roman alphabet that either start with "con" or contain "com" or end with "ant".

(c) The set of strings over the (lower-case) Roman alphabet that start with "con", contain "com" and end with "ant".

(d) $\{\alpha\beta \in \{0,1\}^* \mid \nu_0(\alpha)$ is odd and $\nu_1(\beta)$ is even$\}$, where $\nu_a(\gamma)$ stands for the number of occurrences of the terminal symbol a in the string γ.

9. Prove or disprove the following assertions. Here A is an arbitrary language over the alphabet Σ.

(a) $(A^*)^2 = A^*$. **(b)** $(A^*)^2 = (A^2)^*$. **(c)** $(A^*)^* = A^*$.

10. (a) Write regular expressions for the languages defined in Exercises 7 and 8.

(b) Design a DFA to accept the language of the regular expression $0^*1^*0^*0$.

(c) Design an NFA with *three* states to accept the language of the regular expression $0^*1^*0^*0$.

11. For each finite automaton M of Exercise 6 construct a regular grammar G with $\mathcal{L}(G) = \mathcal{L}(M)$.

12. The languages of the grammars of Exercise 1(a)–(e) are all regular. However, the given grammars are not always regular.

(a) Determine which of the given grammars is/are regular. If a given grammar G is not regular, write a regular grammar with language $\mathcal{L}(G)$.

(b) Convert the regular grammars of Part (a) to equivalent finite state machines.

13. (a) Prove that if L is a regular language, then so also is its complement $\overline{L} := \Sigma^* \setminus L$.

(b) Prove that if L_1 and L_2 are regular languages, then so also is their intersection $L_1 \cap L_2$.

* **(c)** Prove that if L is a regular language, then so also is its reverse $L^R := \{\alpha^R \mid \alpha \in L\}$.

14. Prove that the following languages are not regular:

(a) $\{0^i 1^j \mid i \neq j\} \subseteq \{0,1\}^*$.

(b) $\{\alpha\alpha^R \mid \alpha \in \{0,1\}^*\}$.

(c) $\{\alpha\alpha \mid \alpha \in \{0,1\}^*\}$.

(d) $\{\alpha \in \{0,1\}^* \mid \alpha$ has equal number of 0's and 1's$\}$.

(e) $\{a^{k^2} \mid k \in \mathbb{N}\} \subseteq \{a\}^*$.

15. A context-free grammar G is called ambiguous if some string $\alpha \in \mathcal{L}(G)$ has two (or more) different parse trees.

(a) Show that the grammar of Exercise 1(f) is ambiguous.

* **(b)** Write unambiguous grammars for the languages of Exercise 3(f), 3(g) and 3(h).

16. Let L be a regular language. The minimum number of states a DFA with language L has is called the state complexity of L. Let us denote this by $C(L)$. Note that $C(L)$ is a property of L and not of any particular DFA realizing L.

(a) Let $L \neq \emptyset$ be a finite language in which the biggest length of a string is l. Prove that $C(L) \geqslant l + 2$. Find an example for which $C(L) = l + 2$.

(b) Find an example of an infinite regular language L for which $C(L) = 1$.

(c) Find an example of an infinite regular language L for which $C(L) = 2$.

(d) For any regular language L show that $C(L) = C(\overline{L})$.