CS30053 Foundations of Computing, Autumn 2004

End-Semester Examination

Total points: 50 | November 27, 2004 | Total time: 3 hours

Answer any four parts of Question 1 *and* any three of the remaining four Questions (2–5). You may get a bonus if you attempt more than four parts of Question 1. There is no bonus for solving all of the four Questions 2–5.

This test is open-notes. You may bring any amount of hand-written material.

1. Which of the following are true? No credits will be given without proper justifications. Attempt any four. **(5×4)**

(a) Let $f_1(n), f_2(n), g_1(n), g_2(n)$ be real-valued functions of natural numbers. Suppose also that $f_1(n) = \Theta(g_1(n))$ and $f_2(n) = \Theta(g_2(n))$. Then $f_1(n) + f_2(n) = \Theta(g_1(n) + g_2(n))$.

(b) The number a_n of strings of length n over $\{0, 1\}$, that contain 01 as a substring, satisfies the strict inequality $a_n < (n-1)2^{n-2}$ for all $n \geqslant 4$.

(c) Every string over $\{0, 1\}$ containing equal number of 0's and 1's belongs to the language of the regular expression ((01)∪(10))*.

(d) Let $N = (S, \Sigma, s, F, \delta)$ be an NFA. Consider the NFA $N' = (S, \Sigma, s, S \setminus F, \delta)$ obtained from N by making the final states of N non-final and the non-final states final. Then $\mathcal{L}(N') = \overline{\mathcal{L}(N)}$.

(e) The language $\{\alpha c \beta \mid \alpha, \beta \in \{a, b\}^*$ and the number of a's in $\alpha =$ the number of b's in $\beta\}$ is regular.

(f) The language $\{\alpha \beta \mid \alpha, \beta \in \{a, b\}^*$ and the number of a's in $\alpha =$ the number of b's in $\beta\}$ is regular.

2. (a) Let $A := (\mathbb{R} \times \mathbb{R}) \setminus \{(0, 0)\}$, i.e., A is the real plane with the origin removed. Define a relation ρ on A as $(x, y)\,\rho\,(x', y')$ if and only if $x' = cx$ and $y' = cy$ for some non-zero real number c. Show that ρ is an equivalence relation on A. **(5)**

(b) Let $f : \mathbb{Q} \to \mathbb{Q}$ be an injective function with the properties that $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$ for all $x, y \in \mathbb{Q}$. Prove that f is the identity function on $\mathbb{Q}$. **(5)**

3. Let a_n denote the number of strings of length n over $\{0, 1, 2\}$, that do not contain two consecutive 0's.

(a) Deduce a recurrence relation for a_n. Also determine the requisite boundary conditions. **(5)**

(b) Solve the recurrence relation of Part (a) to obtain a closed-form formula for a_n. **(5)**

4. Consider the language $L := \{\alpha \in \{0, 1, 2\}^* \mid \alpha$ does not contain two consecutive 0's$\}$.

(a) Describe a regular grammar for L. **(5)**

(b) Design a DFA to accept L. **(5)**

5. Consider the context-free grammar $G = (V, \Sigma, S, P)$, where $V = \{S, A, B, a, b\}$, $\Sigma = \{a, b\}$, and P consists of the following productions:

$$\begin{array}{lcl} S & \to & \lambda \mid ASB, \\ A & \to & a, \\ B & \to & \lambda \mid b \mid bb. \end{array}$$

(a) Describe $\mathcal{L}(G)$ in English. **(5)**

(b) Prove that $\mathcal{L}(G)$ is not regular. **(5)**

Dr. Abhijit Das, Dept. of Computer Science & Engineering, IIT Kharagpur, India