CS30053 Foundations of Computing, Spring 2004

Class test 1

Total points: 25 | August 26, 2004 | Total time: $1+\epsilon$ hour

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1. Which of the following statements is/are true? (No credits will be given without proper justifications.) **(2 × 7)**

(a) $((p \to q) \wedge \neg p) \to \neg q$ is a tautology.

(b) $((p \to q) \wedge \neg q)$ is equivalent to $\neg(p \vee q)$.

(c) $\forall x \in \mathbb{R}\ \exists m \in \mathbb{Z}\ \exists n \in \mathbb{Z}\ \Big[m < x < n \Big]$.

(d) $\forall x \in \mathbb{R}\ \exists m \in \mathbb{N}\ \exists n \in \mathbb{N}\ \Big[m < x < n \Big]$.

(e) $\forall x \in \mathbb{R}\ \Big[\lceil \lfloor x/2 \rfloor /3 \rceil = \lfloor \lceil x/2 \rceil /3 \rfloor \Big]$.

(f) The function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(n) := n^2 + 2n + 3$ for all $n \in \mathbb{N}$ is injective.

(g) The function $g : \mathbb{Z} \to \mathbb{Z}$ defined by $g(n) := n^2 + 2n + 3$ for all $n \in \mathbb{Z}$ is injective.

2. Let A be a finite set of cardinality n. Deduce that:

(a) The total number of anti-symmetric relations on A is $2^n 3^{n(n-1)/2}$. **(3)**

(b) The total number of relations on A, that are both symmetric and anti-symmetric, is 2^n. **(3)**

3. Let $\mathbb{A}$ be one of the sets $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$. A function $\varphi : \mathbb{A} \to \mathbb{A}$ is called monotonic increasing if $m \leqslant n$ implies $\varphi(m) \leqslant \varphi(n)$. It is called strictly monotonic increasing if $m < n$ implies $\varphi(m) < \varphi(n)$.

(a) Show that a strictly monotonic increasing function is injective. **(2)**

(b) Give an example of a function $\mathbb{N} \to \mathbb{N}$ that is injective but not strictly monotonic increasing. **(3)**

(c) [Bonus] Let $f : \mathbb{N} \to \mathbb{N}$ be an injective function with the property that $|f(n+1) - f(n)| \leqslant 1$ for all $n \in \mathbb{N}$. Show that f is strictly monotonic increasing. **(5)**

Dr. Abhijit Das, Dept. of Computer Science & Engineering, IIT Kharagpur, India