1. Prove that a TM with a two-dimensional (semi-infinite in both directions) tape is equivalent to a TM.

2. A queue automaton is like a PDA with the only exception that the external memory is organized (and accessed) as a queue. Prove that queue automata are equivalent to Turing machines.

3. Let N be a total NTM. The running time of N expressed in terms of the length n of the input is the height of the tree of configurations of N on an input of length n. Determine the running time of the NTM for accepting composite integers.

4. Design an NTM to accept the following language.

$$\{\ ww^R w \mid w \in \{a, b\}^*\ \}$$

5. Let G be a simple undirected graph with n vertices. Let $M \in \{0, 1\}^*$ denote the string of length n^2 storing the adjacency matrix of G in the row-major order. G is encoded as the string $1^n\#M \in \{0, 1, \#\}^*$. Design an NTM that, given G encoded as a string in this manner, determines whether G contains a Hamiltonian cycle. What is the (worst-case) running time of your NTM as a function of n. This running time should be a polynomial in n.

6. A TM M has a two-way infinite tape. Initially, all cells on the tape are blank. Only one cell is storing the symbol #. The head of M is pointing to a blank. The task of M is to locate the cell storing #. Propose a strategy for doing this

   (a) if M is an NTM, and
   (b) if M is a DTM.
   (c) Compare the nondeterministic and deterministic running times.