More on the language

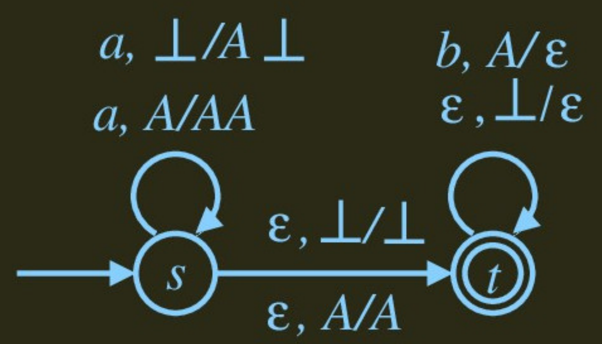
$$\text{cyclicshift}(L) = \{\ yx \mid xy \in L\ \}$$

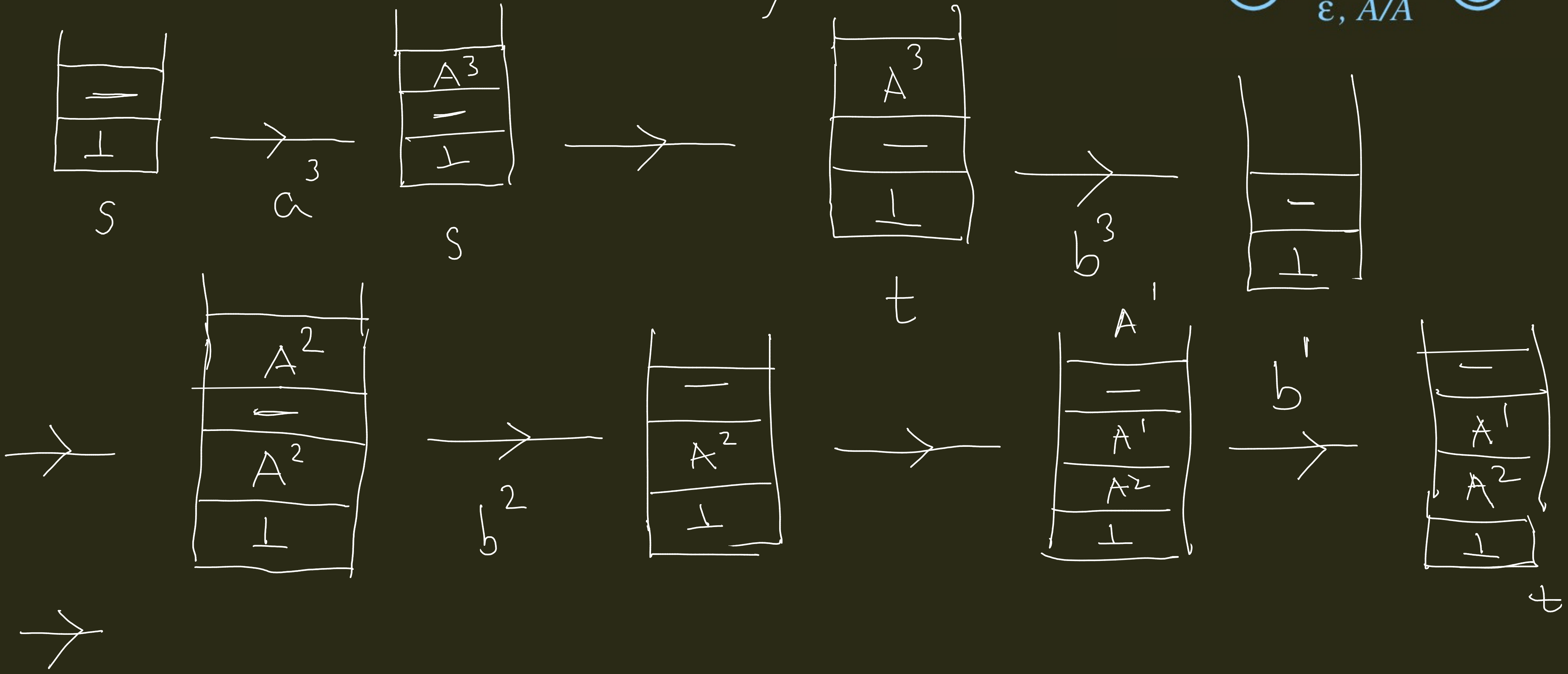
1 2 3 3 2 1

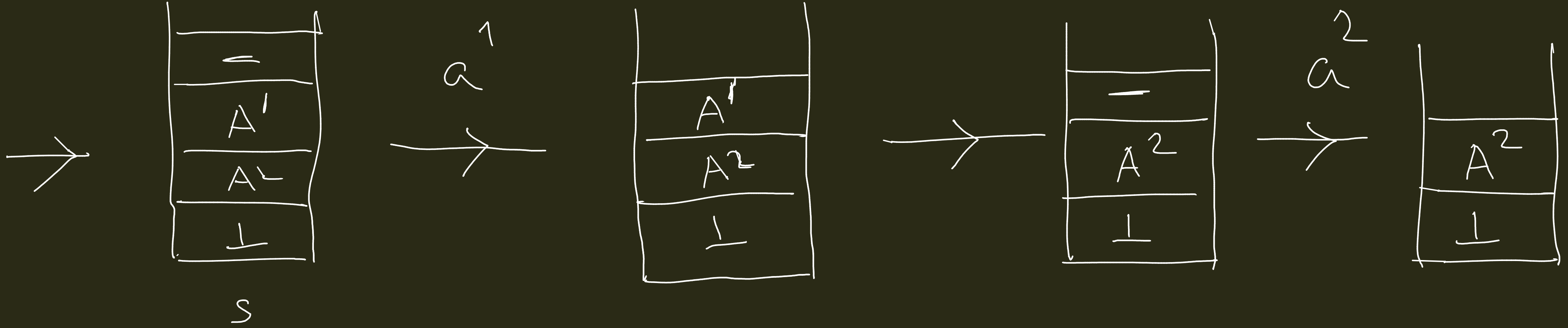
$\underbrace{a}_{x}\underbrace{aabbb}_{y}$

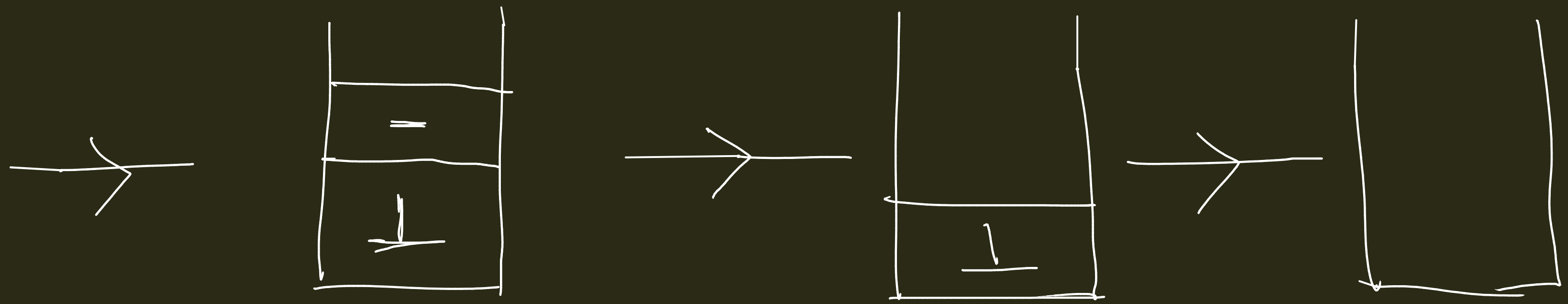
3 3 2 1 1 2

$\underbrace{abbb}_{y}\underbrace{aa}_{x}$









$$\underbrace{ab}_{x}\,\underbrace{baab}_{y}$$

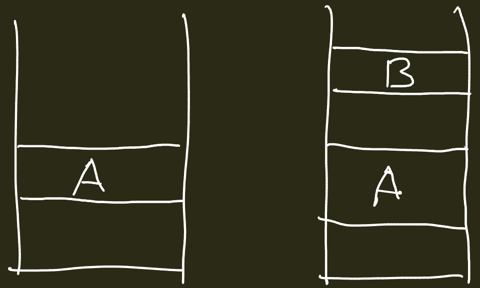
Exercise

$$S \to \epsilon \mid aSb \mid bSA \mid SS$$

1. Let M be a PDA that never pops from its stack. M accepts by final state. Prove that $\mathscr{L}(M)$ is regular.

$$M = (Q, \Sigma, \Gamma, \perp, \delta, q, F)$$

$$\rightarrow \quad N = (Q', \Sigma, \delta', q', F')$$

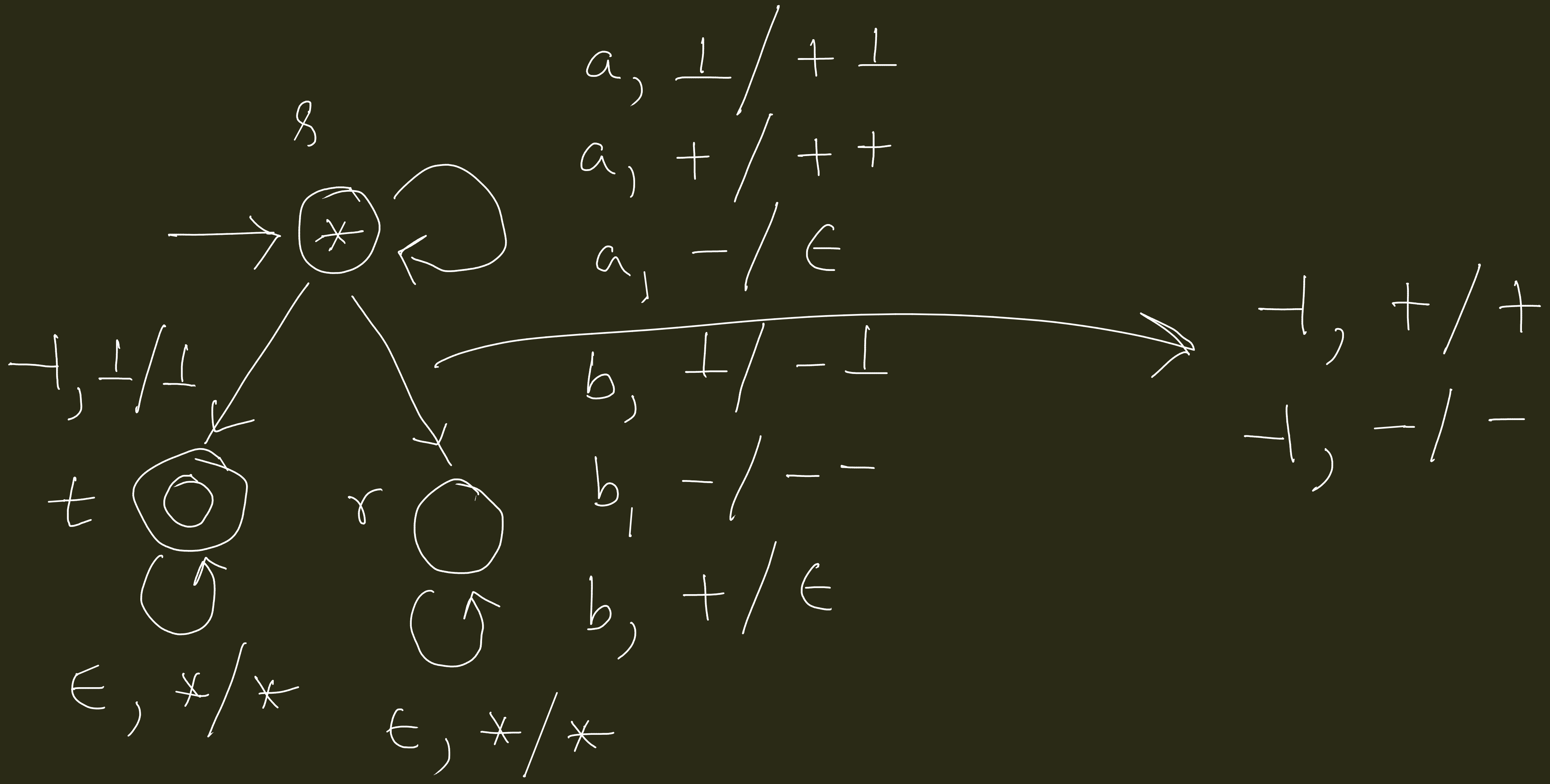


$$Q' = Q \times \Gamma \qquad q' = (q, \perp)$$

$$F' = F \times \Gamma$$

$$\delta'((p, A), a) = \{ (q, B) \mid ((p, A, a), (q, B\gamma)) \in \delta \}$$

2. Prove that the language { x ∈ {a,b}* | #a(x) = #b(x) } is a DCFL.



3. It is known that a deterministic CFL is also unambiguous. Give an example of a UCFL that is not a DCFL. Prove unambiguity, and intuitively justify the non-deterministic property of your language.

$$\mathrm{DCFL} \underset{\neq}{\subset} \mathrm{UCFL}$$

defined by m/cs

defined by grammars

{palindromes over {a, b}}

4. Prove that the following grammar for all balanced-parentheses expressions is unambiguous.

$S \longrightarrow (S)S \mid \varepsilon$

Assume two decompositions

$$w = (x)y = (u)v$$

$x \neq u$

$y \neq v$

$(\text{---} x \text{---}) \text{---} y \text{---}$

$(\text{---} u \text{---}) \text{---} v \text{---}$

Arrive at a contradiction

5. Consider the language $L = \in \{a,b\}^* \mid \#a(x) = \#b(x) \}$.

(a) Prove that the language of the grammar $S \longrightarrow aSbS \mid bSaS \mid \varepsilon$ is L.
(b) Prove that the grammar of Part (a) is ambigous.
(c) Design an unambiguous grammar for L.

Use Exercise 4