1. Consider the language $L_1 = \{ x \in \{a,b\}^* \mid \#a(x) = 2 \times \#b(x) \}$.

(a) Design a CFG for L_1.

P/D: $S \to \epsilon \mid aabS \mid aaSb \mid aSab \mid Saab \mid abaS \mid \dots \mid SS$

aaabbbaaa cannot be generated

$$s(x) = \#a(x) - 2 \times \#b(x)$$

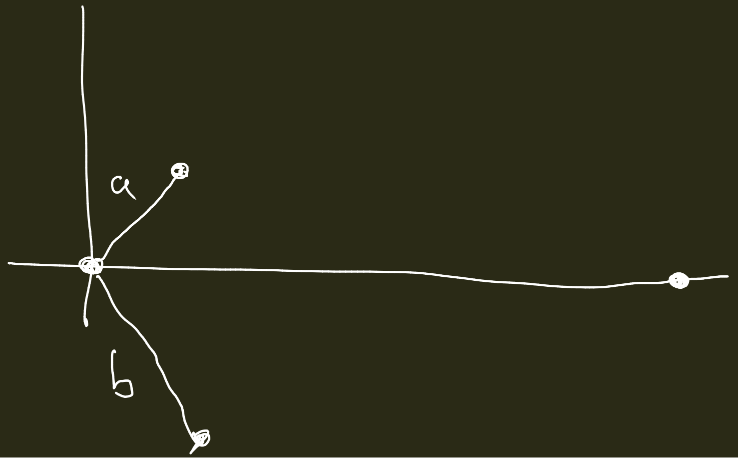
$S \to \{ x \mid s(x) = 0 \}$

$A \to \{ x \mid s(x) = 1 \}$

$B \to \{ x \mid s(x) = -1 \}$

Rules for S

$$S \to \epsilon \mid aB \mid bAA$$

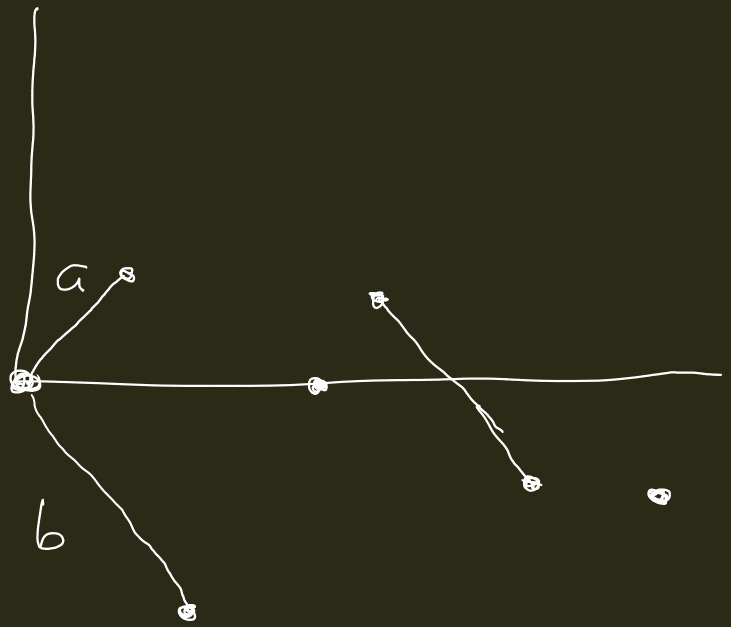


Rules for A

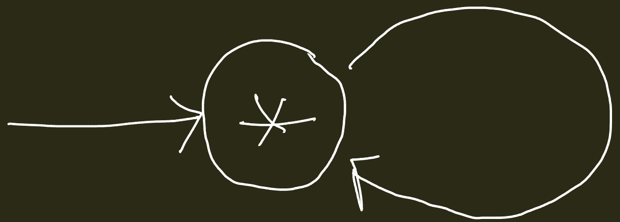
$$A \to aS \mid bAAA$$

Rules for B

$$B \to bA \mid aBB \mid aSbS$$



(b) Convert the CFG of Part (a) to a PDA for L_1.



$a, a / \epsilon$

$b, b / \epsilon$

$\epsilon, S / \epsilon$ $\quad \epsilon, S / aB$ $\quad \epsilon, S / bAA$

$\epsilon, A / aS$ $\quad \epsilon, A / bAAA$

$\epsilon, B / bA$ $\quad \epsilon, B / aBB$ $\quad \epsilon, B / aSbS$

(c) Design a PDA for L_1 from the scratch.

$a, \bot / +\bot$

$a, + / ++$

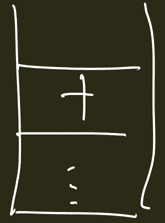
$a, - / \epsilon$

$b, \bot / --\bot$

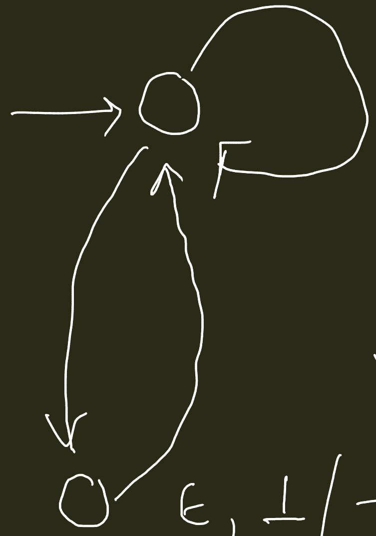
$b, - / ---$

$\epsilon, \bot / \epsilon$

b



$b, + / \epsilon$



$\epsilon, \bot / -\bot$

$\epsilon, + / \epsilon$

$b, \bot / -\bot$

$b, - / --$

$b, + / \epsilon$

2. Design a PDA for the language $\{ x \in \{a,b\}^* \mid \#b(x) \leqslant \#a(x) \leqslant 2 \times \#b(x) \}$.

Extra transition added in red.

$$k = \#b(x)$$

$$k \leq \#a(x) \leq 2k$$

$$\#a(x) = k + l = l + (k-l) + l = 2l + (k-l)$$

$$0 \leq l \leq k$$

$$0 \leq k - l \leq k$$

3. Design a PDA for the language

$\{a,b,c\}^* - \{ a^n b^n c^n \mid n \geq 0 \}$.

1. strings containing ba
2. — " — cb
3. — " — ca
4. $\underline{a^i b^j} c^k$ s.t. $i \neq j$
5. $a^i \underline{b^j c^k}$ s.t. $j \neq k$

4. Let L be a CFL over some alphabet Γ. Prove that the language

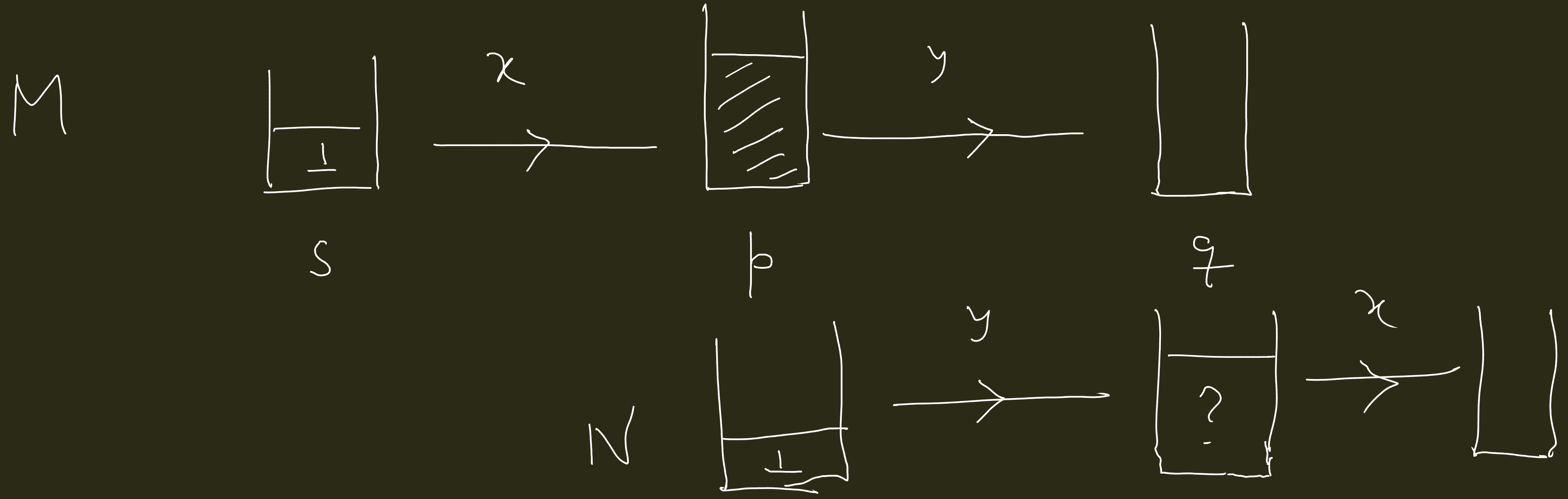
cyclicshift(L) = { yx | xy ∈ L }

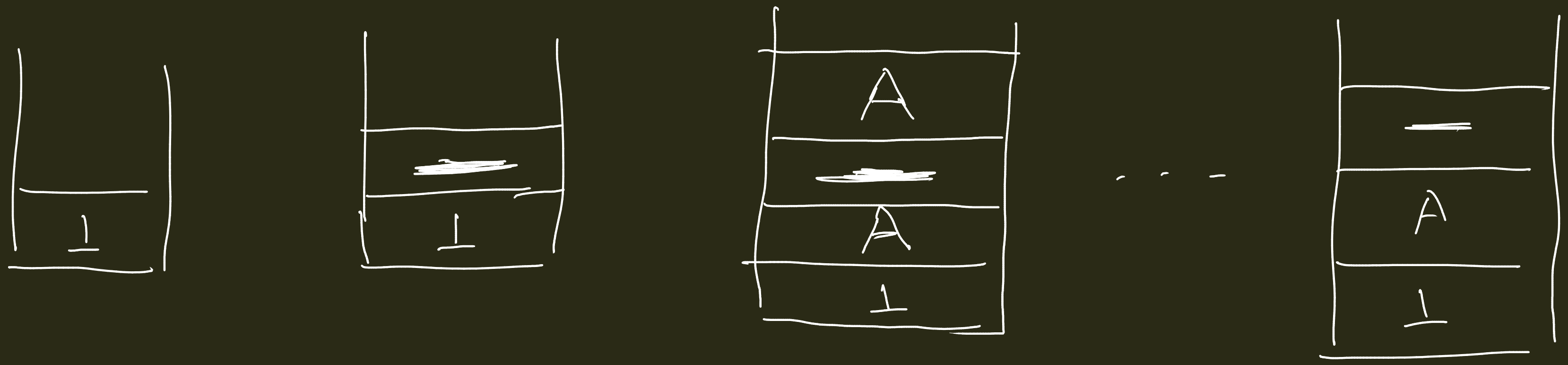
is also a CFL.

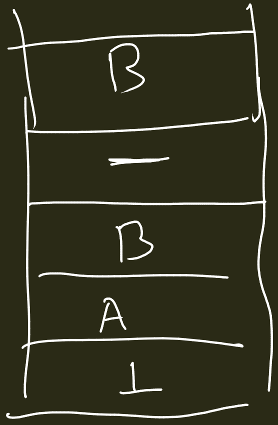
$L = \mathcal{L}(M)$ → accepts by empty stack

$(yx)^r = x^r y^r$

$(xy)^r = y^r x^r$







HW:

$S \to \epsilon \mid aSb$

aaabbb

abbbaa

$S \to \epsilon \mid aSa \mid bSb$