1. Consider the language

$A_k = \{w \in \{0,1\}^* \mid$ The last k positions in w have at least one 1$\}$.

(a) Prove that any DFA accepting A_k must have at least k + 1 states.
(b) Design a DFA with k + 1 states for A_k.

(a) Suppose that a DFA with $\leq k$ states accepts A_k.

$\leq k-1$ final states

1 1

10 10

10^2 100

10^{k-1} $1000\cdots0$

$$\hat{\delta}(s, 10^i) = \hat{\delta}(s, 10^j)$$

$i > j$ Append 0^{k-1-j}

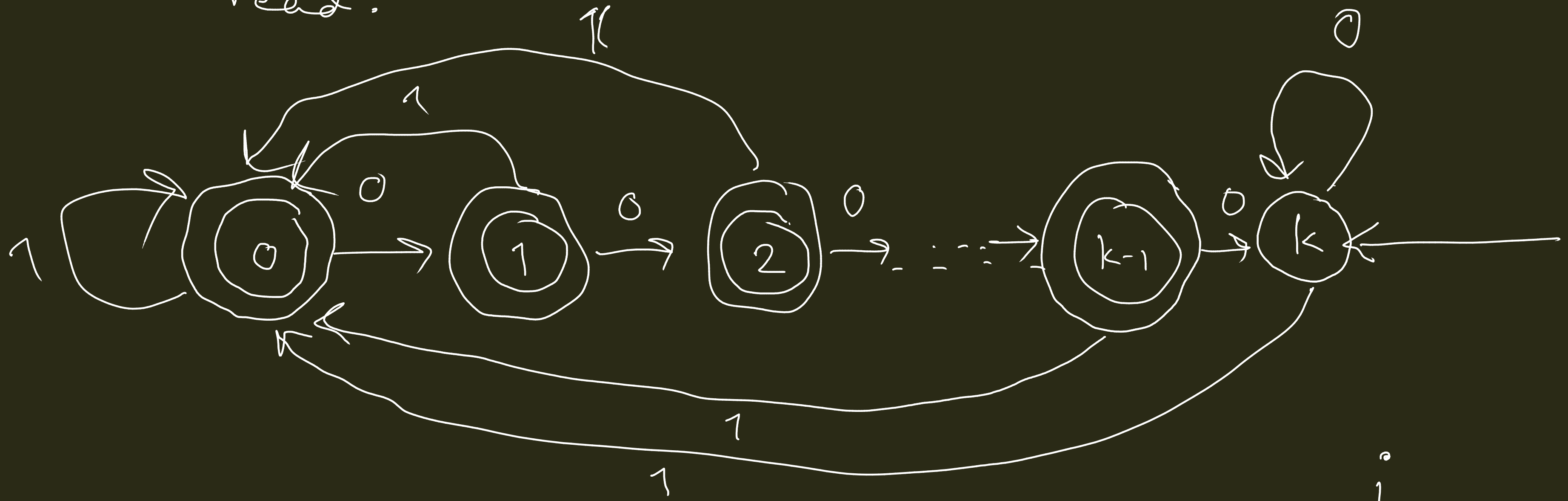
$k-1+(i-j)$

$\geq k-1+1 = k$

$10^{k-1-j+i}$ $\to$ not in A_k

10^{k-1} $\to$ in A_k

(b) It suffices to remember when the last 1 was read.



State i $\rightarrow$ last read part 10^i

2. Let E_k = { $w \in \{0,1\}^*$ | the last k positions of w contain exactly one 1 }. Prove/Disprove: Any DFA for E_k contains at least 2^k states.

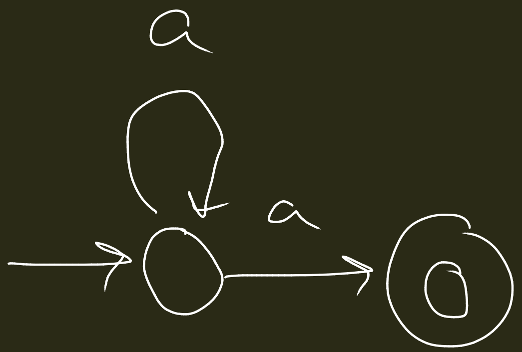
$$\binom{k+1}{2} + 1 \quad \text{states}$$

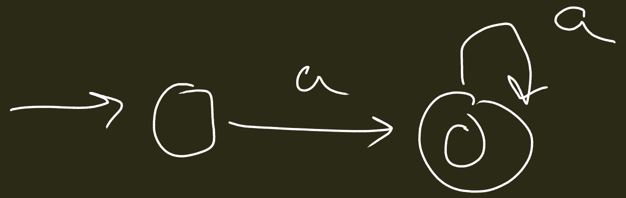
3. Prove that the minimal NFA for a regular language is not necessarily unique.

a^*

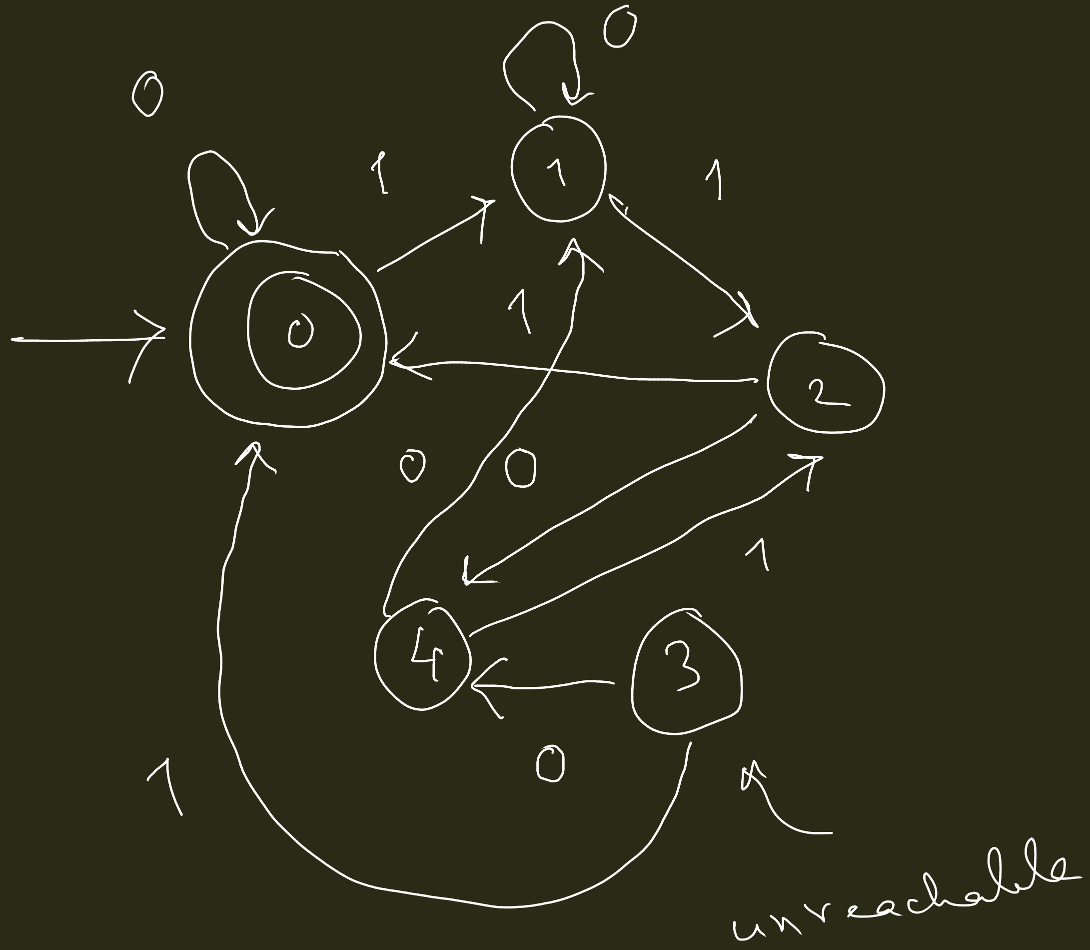
$a^+ \equiv aa^*$

↳ must have 2 states





4. Consider the DFA with $\Sigma = \{0,1\}$, $Q = \{0, 1, 2, 3, 4\}$, $s = 0$, $F = \{0\}$, and $\delta(q,a) = (q^2+a) \bmod 5$. Minimize the DFA using the algorithm disscussed in the class.



0			
X	1		
X	X	2	
X	.	X	4

1, 4 eqvt.

5. Consider the regular language $L = \mathscr{L}(a^*b^*+b^*a^*)$ over $\{a,b\}$. Design the minimal DFA for L. From this DFA, identify the equivalence classes of $\equiv_L$.

Partition in

$\{\epsilon\}$

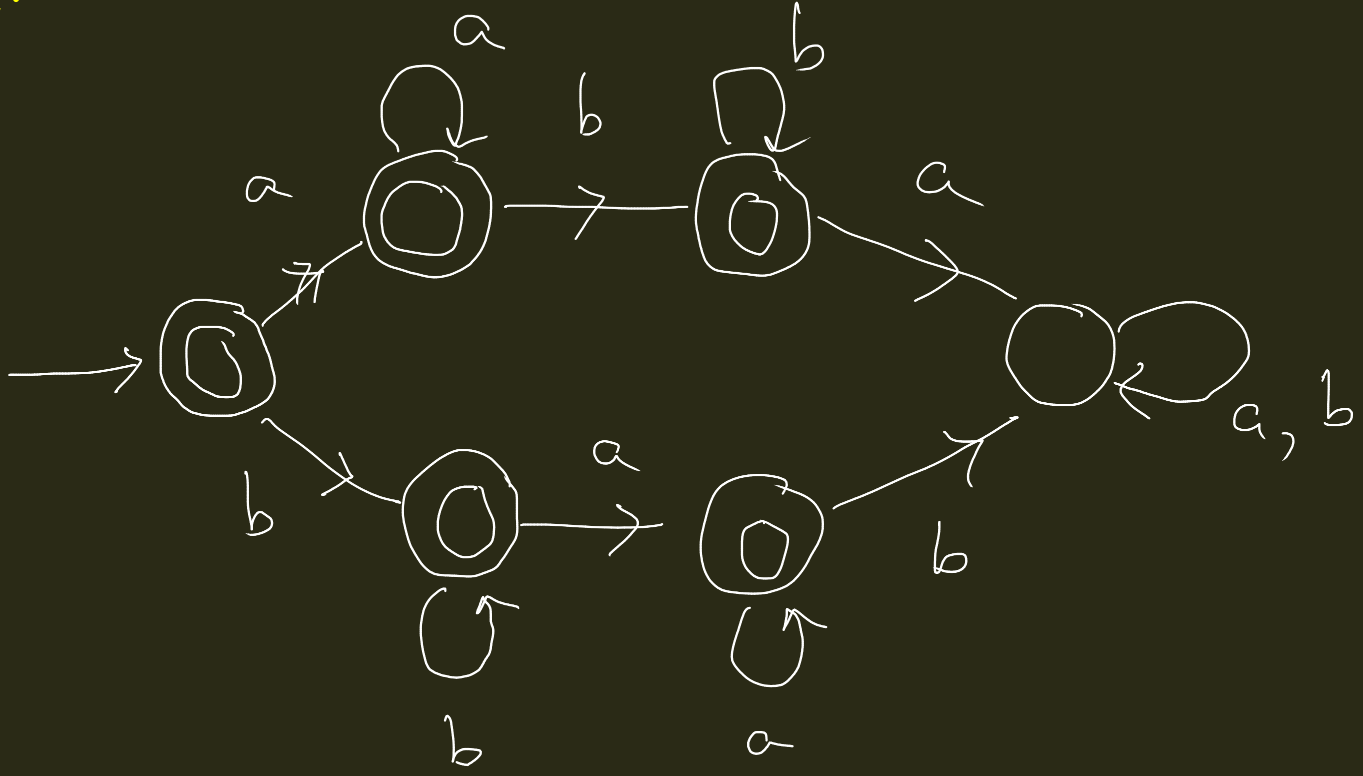
$\mathcal{L}(a^+)$

$\mathcal{L}(a^+b^+)$

$\mathcal{L}(b^+)$

$\mathcal{L}(b^+a^+)$

$\overline{L}$



6. Use the Myhill–Nerode theorem to prove that the language

EQ = { w ∈ {a,b}* | #a(w) = #b(w) }

is not regular.

TST $\equiv_{EQ}$ is not of finite index

$a^0, a^1, a^2, \ldots$ all belong to distinct equivalence classes

$a^i, a^j \quad i \neq j$

$a^i b^i \in EQ, \quad a^j b^i \notin EQ.$

Identify the equivalence classes

$$\delta(x) = \#a(x) - \#b(x) \qquad 0, \pm 1, \pm 2, \pm 3, \ldots$$

Each equivalence class consists of all strings of the same surplus

7. Use the Myhill–Nerode theorem to prove that the language

$\{ w \in \{a,b\}^* \mid \#a(w) - \#b(w) \text{ is a multiple of } 5 \} = L$

is regular.

TST

$\equiv_L$ has a finite index

Equivalence classes

$x \longrightarrow s(x) \bmod 5 \in \{0,1,2,3,4\}$

$x, y \longrightarrow$ with the same surplus

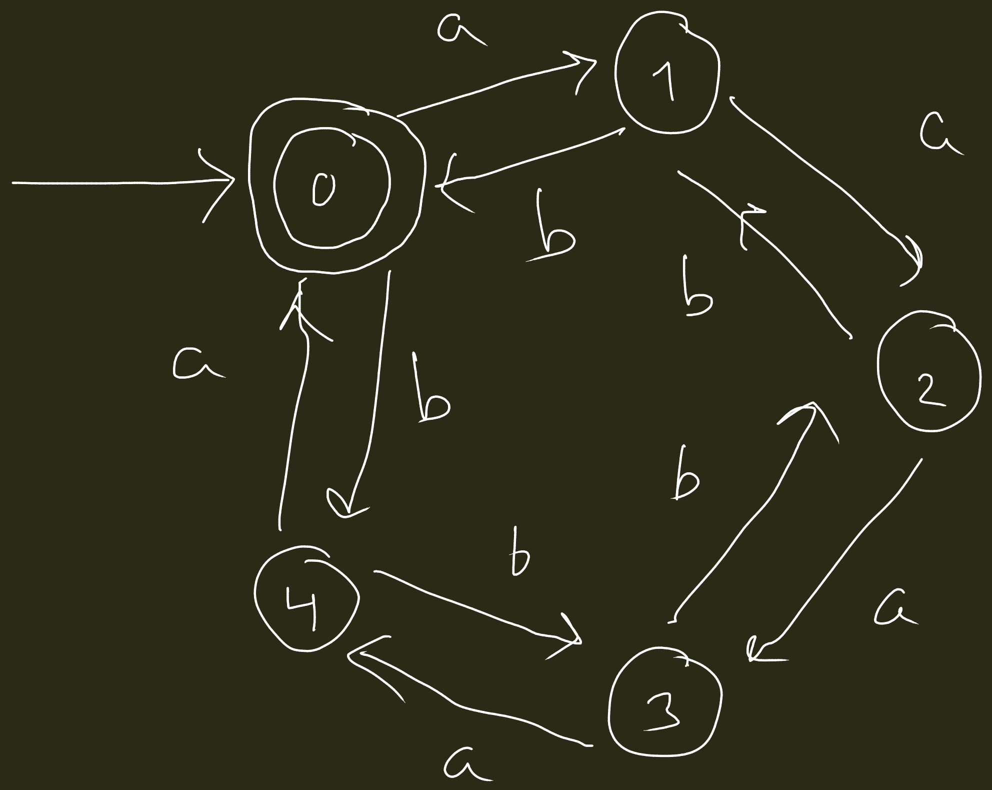
$s(x) = s(y)$

For all z, $s(xz) = s(x) + s(z) = s(y) + s(z)$

$= s(yz)$

$s(x) \neq s(y) \Rightarrow x \not\equiv y$

Take $z = a^{(5 - s(x)) \bmod 5}$


0
1
2
3
4
a
b