1. Consider the language

$$L = \{a^n b^n c^n \mid n \geq 0\}$$

over the alphabet {a,b,c}. Characterize all the strings in the complement ~L.

- strings containing ba, ca or cb
- strings $a^i b^j c^k$ with $i \neq j$ or $j \neq k$

2. Give an example of two infinite languages A and B over the alphabet {a,b} such that $A \cap B = \varnothing$ and $AB = BA$, or prove that no such languages exist.

$$A = \{ x \in \{a,b\}^* \mid |x| \text{ is even} \}$$

$$B = \{ x \in \{a,b\}^* \mid |x| \text{ is odd} \}$$

$$AB = BA = B$$

3. Give an example of a language L (over any alphabet of your choice) such that

$\sim(L^*) = (\sim L)^*$,

or prove that no such language can exist.

For any L, $L^0 = \{\epsilon\}$

True even for $L = \emptyset$

$\emptyset^* = \{\epsilon\}$

contains $\epsilon$

contains $\epsilon$

does not contain $\epsilon$

False (No such L can exist)

4. Let B be a language over some alphabet. We call B transitive if $BB \subseteq B$. We call B reflexive if $\varepsilon \in B$. Prove that for any language A, the smallest transitive and reflexive language containing A is $A^*$.

- $A^*$ is transitive and reflexive $\quad A^*A^* = A^*$

  $\epsilon$ is in $A^*$

- Suppose B is a transitive and reflexive language containing A

  We show that $A^* \subseteq B$.

$A \subseteq B$ $\quad A^0 = B^0 = \{\epsilon\} \subseteq B$

$AA \subseteq BB \subseteq B$ $\quad A^2 \subseteq B$ $\quad A^n \subseteq B$

$A^3 = A^2 A \subseteq BB \subseteq B$ $\quad$ for all $n \geq 0$.

5. Design DFA for the following languages over the alphabet {a, b}.

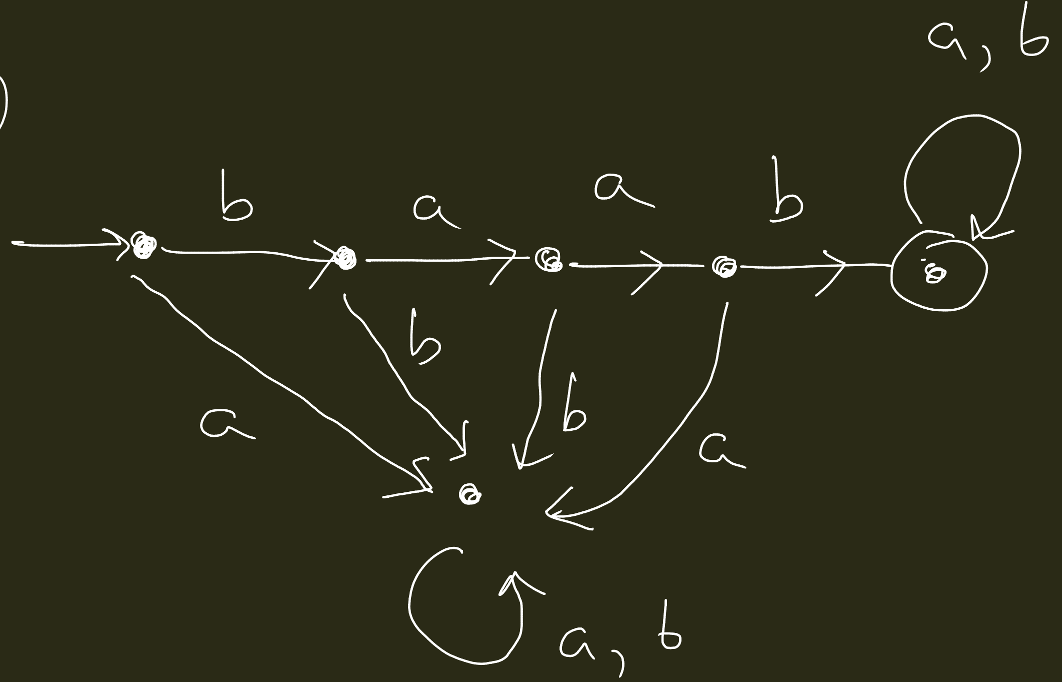
(a) { x | x starts with baab }

(b) { x | x ends with baab }

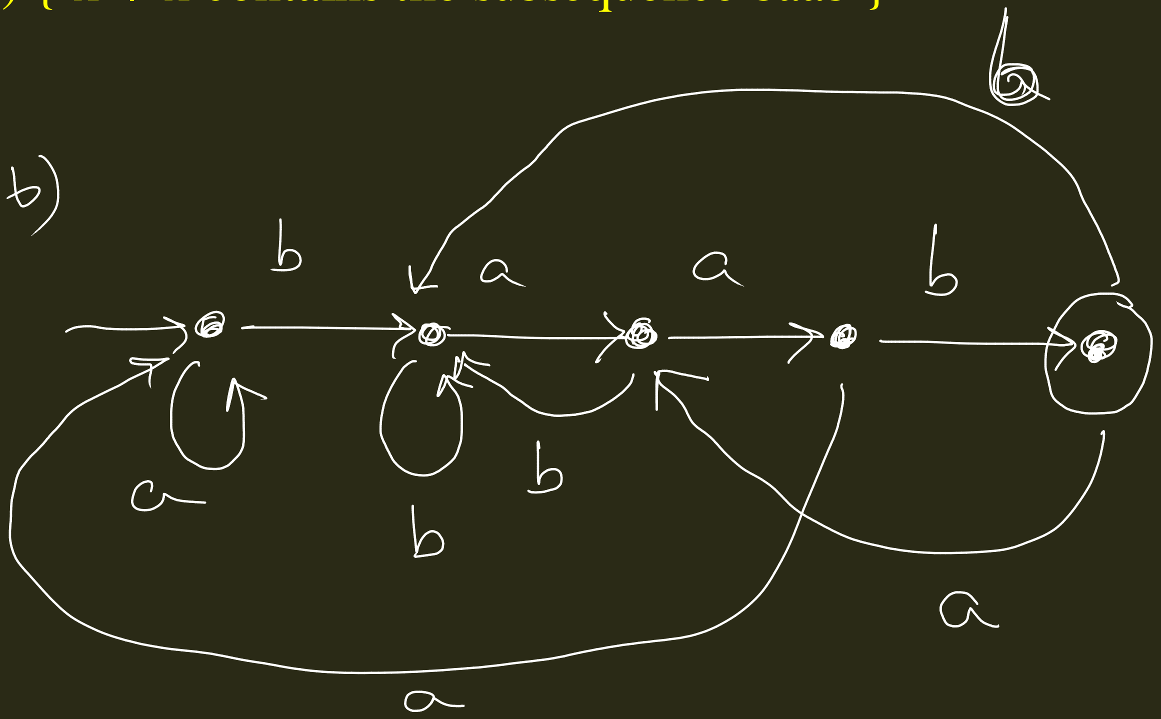
(c) { x | x contains the substring baab }

(d) { x | x contains the subsequence baab }

(a)

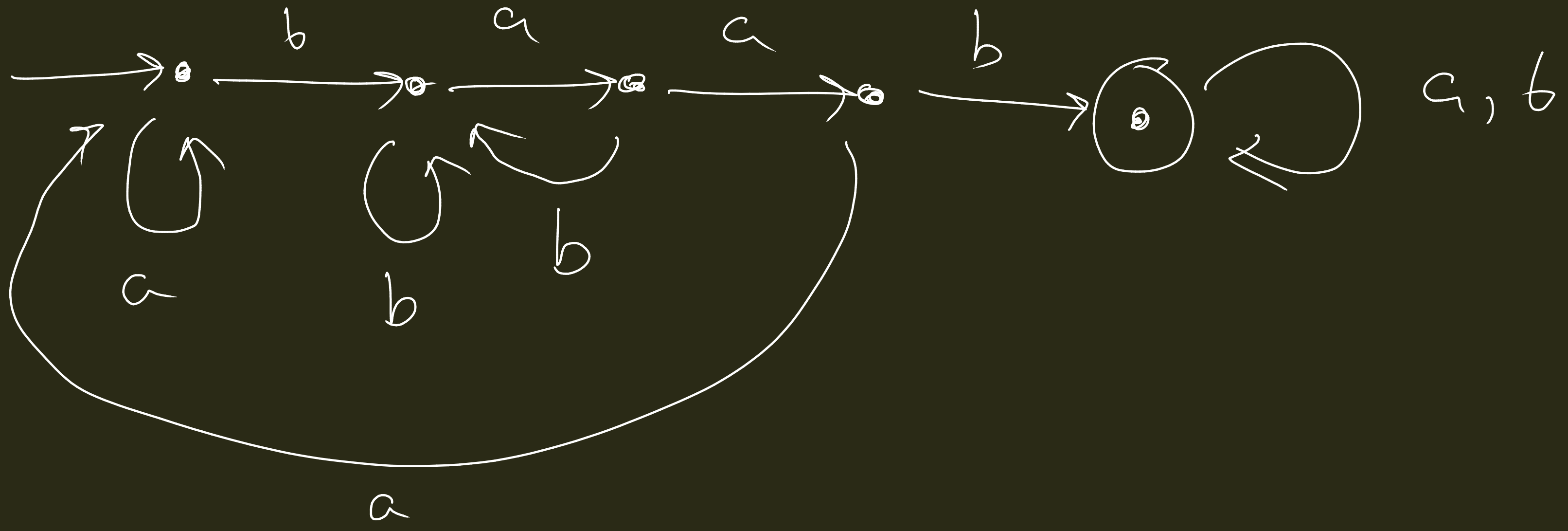



(b)

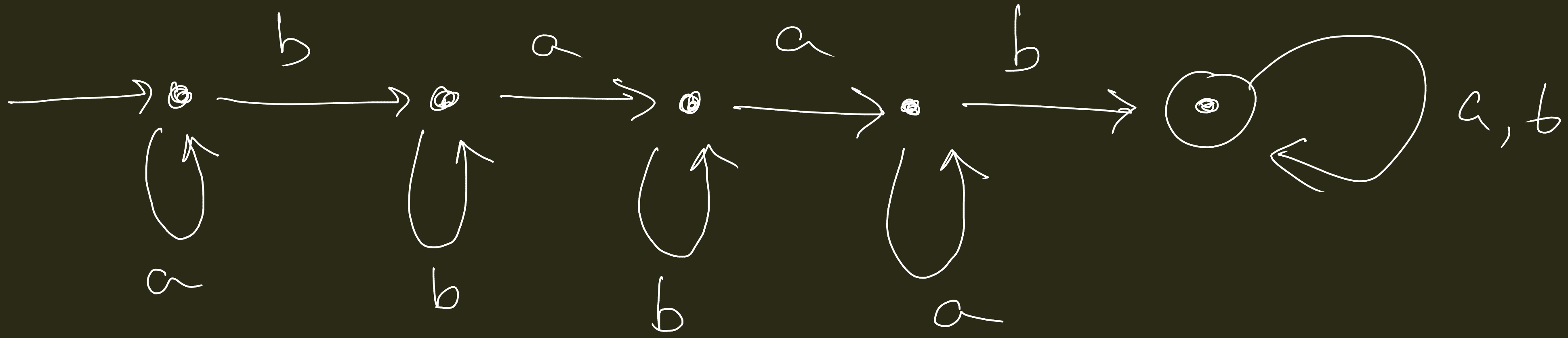



baabaab

(c)




(d)



6. For two languages A and B over the same alphabet, define the language

$A / B = \{x \mid xy \in A \text{ for some } y \in B\}$.

Give examples of infinite languages A and B over the alphabet {a,b} such that:

(a) $A / B = A$

(b) $A / B \neq A$

(b) $A = \{a^i \mid i \geq 0\}$

$B = \{b^j \mid j > 0\}$

$A/B = \phi, A \neq \phi$

P/D: $(A/B)B = A$.

$A = \{a^i \mid i \geq 0\}$

$B = \{a^i b^j \mid i,j \geq 0\}$

$A/B = A$

$(A/B)B = AB \neq A$

$A/B \subseteq A$

↳ consists of prefixes of strings from A

$A \subseteq A/B$

$x \in A$, take $y = \epsilon$

7. Let A be a language over the alphabet {a,b}. Define the language

$B = \{xy \mid xay \in A\}$.

Assume that A is non-empty and not contained in $\{a\}^*$. Prove/disprove: We must have $B \neq A$.

$$A = \{ b a^n \mid n \geq 0 \}$$

$$B = A$$