$$\begin{tikzcd} \text{PDA } M \arrow[rr] \arrow[dr] & & \text{CFG } G \\ & \text{PDA } N \arrow[ur] & \end{tikzcd}$$

M – accepts by both final state
and empty stack

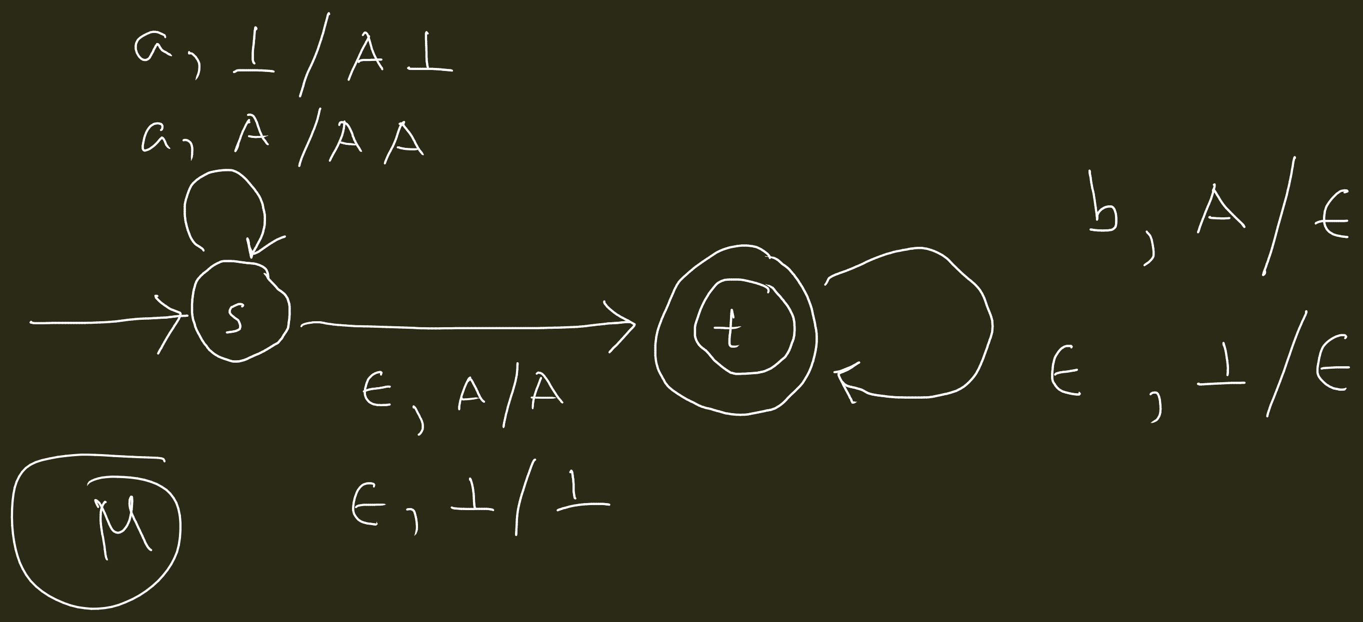
t is the only
final state

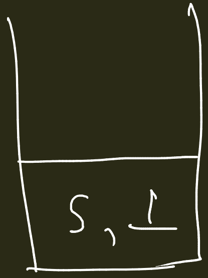
N – only one state
accepts by empty stack

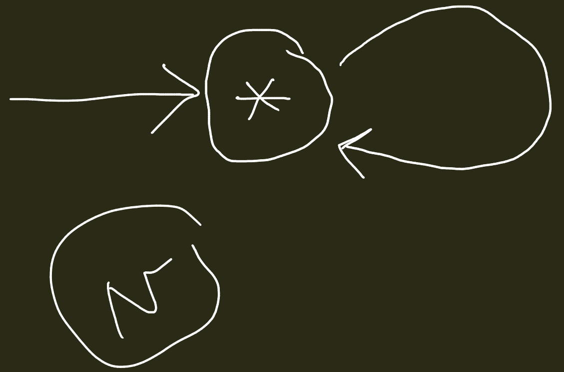
$M \longrightarrow N$

"stores" the state of M in its
stack along with usual stack
symbols of M

$\{ a^n b^n \mid n \geq 0 \}$







$a, (s, \perp) / (s, A)(t, \perp)$

$a, (s, A) / (s, A)(t, A)$

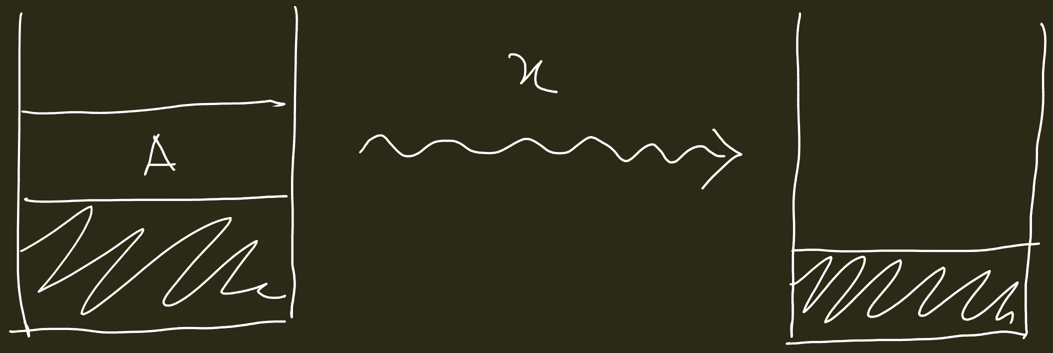
$\epsilon, (s, A) / (t, A)$

$\epsilon, (s, \perp) / (t, \perp)$

$b, (t, A) / \epsilon$

$\epsilon, (t, \perp) / \epsilon$

M



p

q

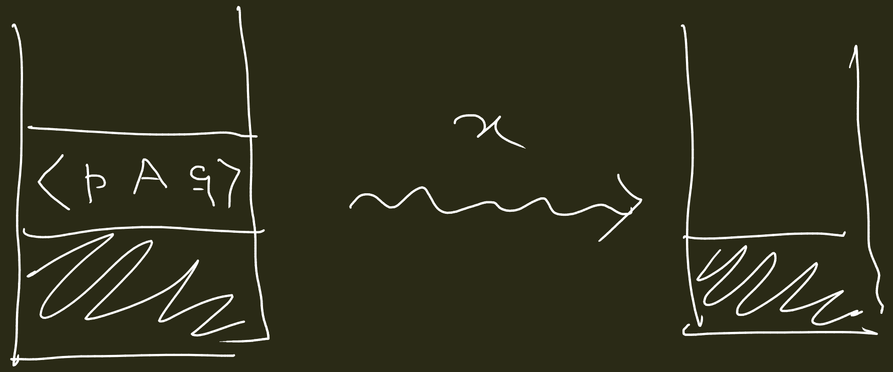
N

$\langle p\ A\ q \rangle$

$Q \times \Gamma \times Q$

$$(p, x\alpha, A\gamma) \overset{*}{\underset{M}{\vdash}} (q, \alpha, \gamma)$$

N

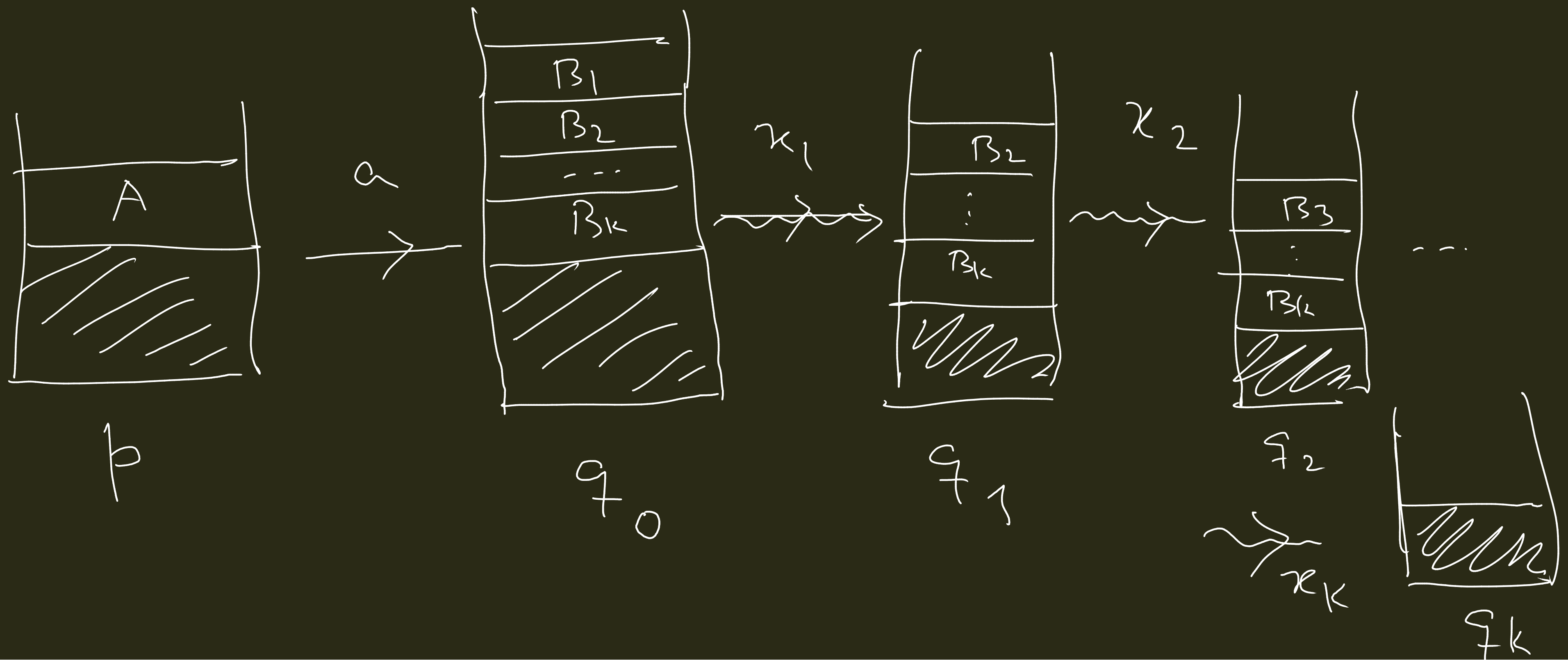


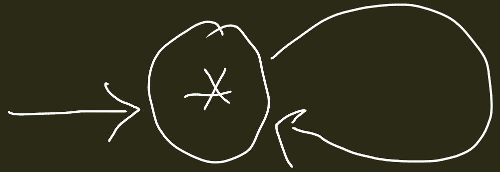
$$(*, x\alpha, \langle p\ A\ q \rangle \delta) \overset{*}{\underset{N}{\vdash}} (*, \alpha, \delta)$$

Initial stack symbol (bottom marker) for N

$$\langle s \perp t \rangle$$

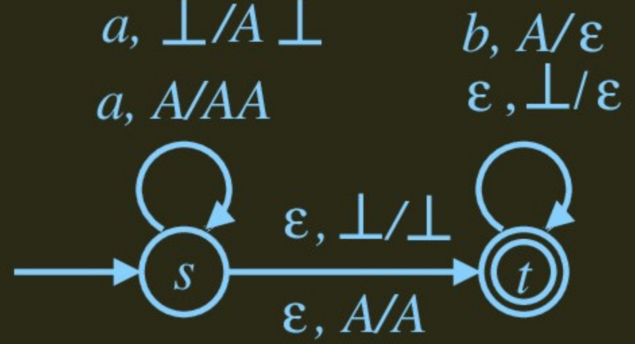
$$((p, a, A), (q_0, B_1 B_2 \dots B_k)) \in \delta_M$$





$$a, \langle p\ A\ q_k \rangle / \langle q_0\ B_1\ q_1 \rangle \langle q_1\ B_2\ q_2 \rangle \cdots \langle q_{k-1}\ B_k\ q_k \rangle$$

N guesses $q_1, q_2, \ldots, q_k$



$((s,a,\perp),(s,A\perp))$
$((s,a,A),(s,AA))$
$((s,\varepsilon,\perp),(t,\perp))$
$((s,\varepsilon,A),(t,A))$
$((t,b,A),(t,\varepsilon))$
$((t,\varepsilon,\perp),(t,\varepsilon))$

$a, \langle s \perp q_2 \rangle / \langle s A q_1 \rangle \langle q_1 \perp q_2 \rangle$

$q_1 \in \{s,t\}, \quad q_2 \in \{s,t\}$

useless

$\langle p\, U\, s \rangle$

$p \in \{s,t\}$

$U \in \{\perp, A\}$

$\langle s\perp t\rangle$ $s, \perp$
$\langle s A t\rangle$ s, A
$\langle t\perp t\rangle$ $t, \perp$
$\langle t A t\rangle$ t, A

useful stack symbols

$((*, a, \langle s\perp t\rangle), (*, \langle s A t\rangle \langle t\perp t\rangle))$
$((*, a, \langle s A t\rangle), (*, \langle s A t\rangle \langle t A t\rangle))$
$((*, \varepsilon, \langle s\perp t\rangle), (*, \langle t\perp t\rangle))$
$((*, \varepsilon, \langle s A t\rangle), (*, \langle t A t\rangle))$
$((*, b, \langle t A t\rangle), (*, \varepsilon))$
$((*, \varepsilon, \langle t\perp t\rangle), (*, \varepsilon))$

Deterministic PDA (DPDA)

(1) (p, a, A)
- There is a unique transition
- There is no such transition
  $\rightarrow$ There must be a transition

$$((p, \epsilon, A), (\dots))$$

(2) A DPDA never halts or gets stuck

(3) $\perp$ is never popped out

(4) A DPDA can accept only by final state

(5) The input is terminated by a spl symbol $\dashv$

$$\mathcal{L}(M) = \{ x \in \Gamma^* \mid (s, x\dashv, \perp) \xrightarrow[M]{*} (f, \epsilon, \gamma\perp) \text{ for some } f \in F \text{ and } \gamma \in \Gamma^* \}$$

$L \subseteq \Sigma^*$ is called a DCFL if $L = \mathcal{L}(M)$ for some DPDA M.

$$\text{DCFL} \subseteq \text{CFL}$$ ← This is a proper inclusion.

Non-determinism imparts better language-recognition power.

<u>Theorem</u>: DCFLs are closed under complement.

$$L = \left\{ a^n b^n \mid n \geq 0 \right\}$$

$$\Sigma = \{a, b\}$$
$$\Gamma = \{A, R, \perp\}$$

Switching final and non-final states does not prove the theorem.

$\left.\begin{array}{l} a, A/RA \\ a, \perp/R\perp \end{array}\right\}$ b after a

$b, A/\epsilon$ — normal

$b, \perp/R\perp$ — more b's than a's

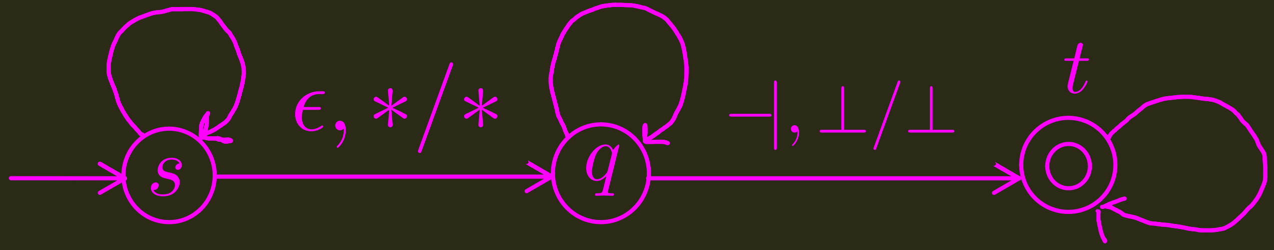
$\dashv, A/RA$ — more a's than b's

$\epsilon, R/R$ — reject loop

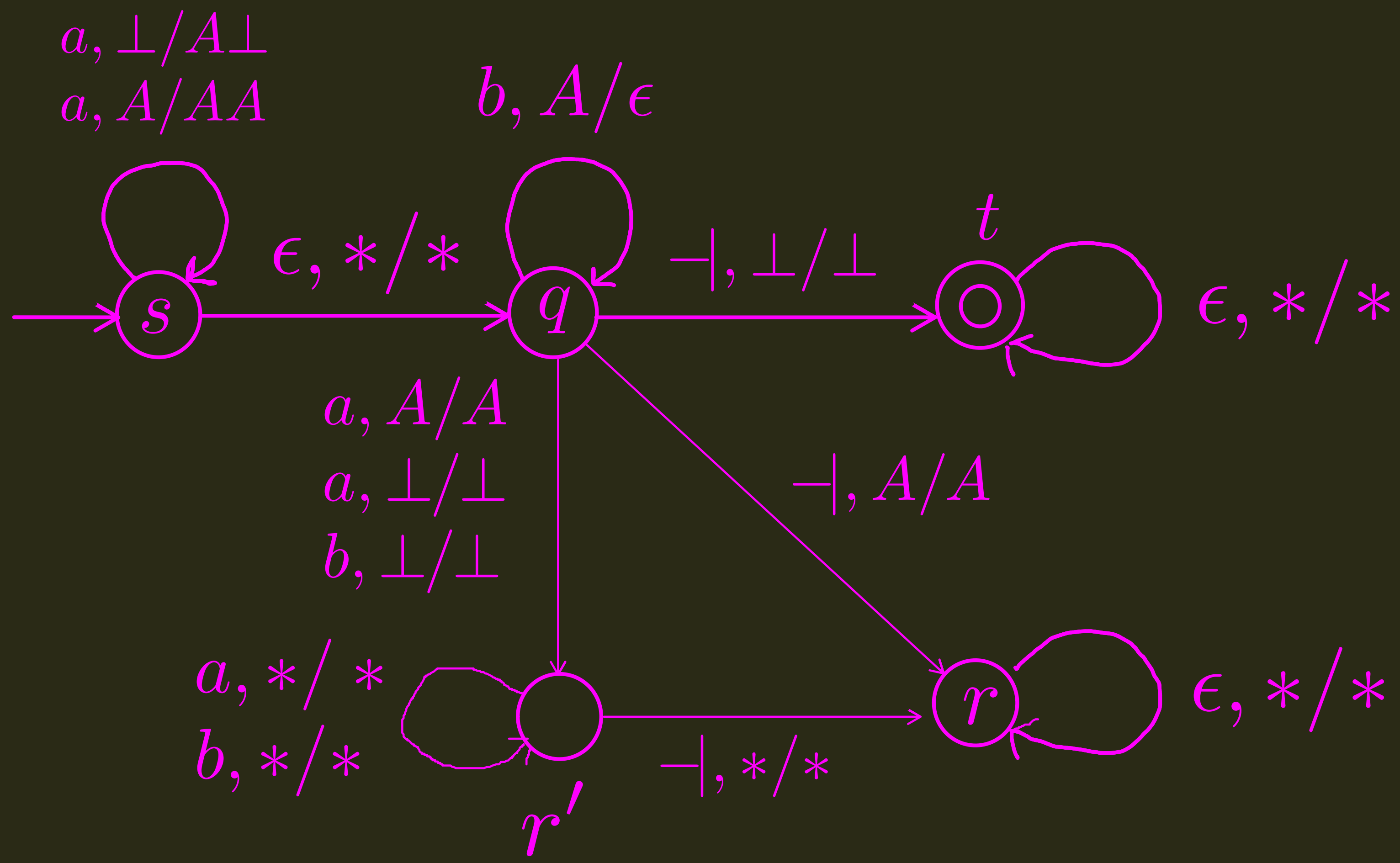
- Make a single reject state r
- How to detect premature reject loops

$a, \perp/A\perp$

$a, A/AA$



$\epsilon, */*$ — accept loop


$a, \perp/A\perp$
$a, A/AA$
$b, A/\epsilon$
$\epsilon, */*$
$\dashv, \perp/\perp$
t
$\epsilon, */*$
s
q
$a, A/A$
$a, \perp/\perp$
$b, \perp/\perp$
$\dashv, A/A$
$a, */*$
$b, */*$
r'
$\dashv, */*$
r
$\epsilon, */*$

Closure properties of DCFL

— closed under ~

— <u>not</u> closed under $\cup$, $\cap$, reversal

Intersection:

$$\{a^m b^m c^n\} \cap \{a^m b^n c^n\}$$

$$= \{a^n b^n c^n\}$$

not even a CFL

Union: $L = \{a^i b^j c^k \mid i \neq j\} \cup \{a^i b^j c^k \mid j \neq k\}$

$\sim L$ is DCFL and no a CFL.

$\sim L \cap \mathcal{L}(a^* b^* c^*)$ is a CFL

$$= \{a^n b^n c^n\}$$

Reversal: $\{b a^i b^j c^k \mid i = j\} \cup \{c a^i b^j c^k \mid j = k\}$