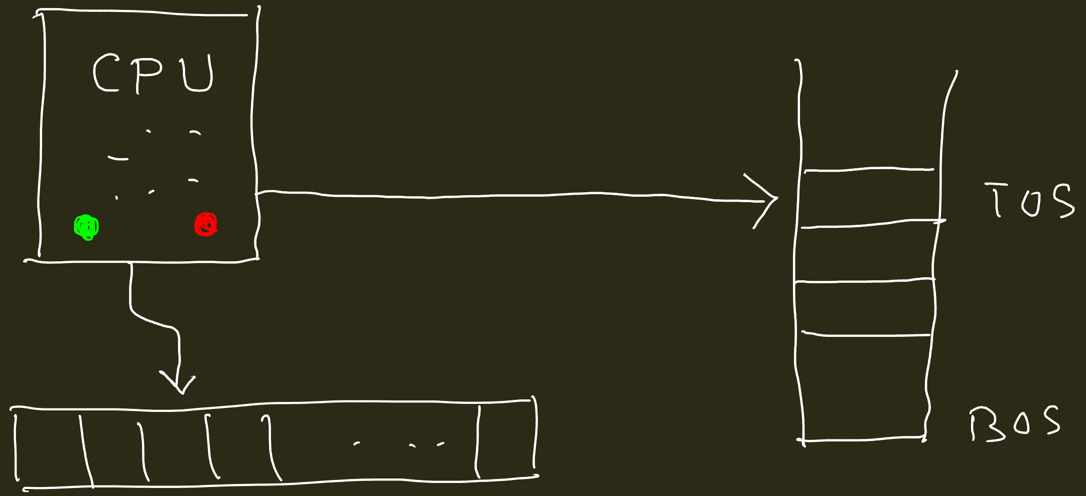
Pushdown Automata (PDA)

pushdown store — stack

deterministic

DPDA



R/O input tape

External memory
stack

- must read the entire input
- transitions depend on three things

1. current state
2. input symbol (allow ϵ)
3. the symbol at the stack top

(a) a "new" state

(b) Replace the top stack by a sequence of stack symbols.

By definition,
PDA are nondeterministic
NPDA

$$M = (Q, \Sigma, \Gamma, \perp, \delta, s, F)$$

Q: a finite set of states

Σ: input alphabet

Γ: stack alphabet

$\perp$: initial bottom marker

δ: transition function

s: the start state

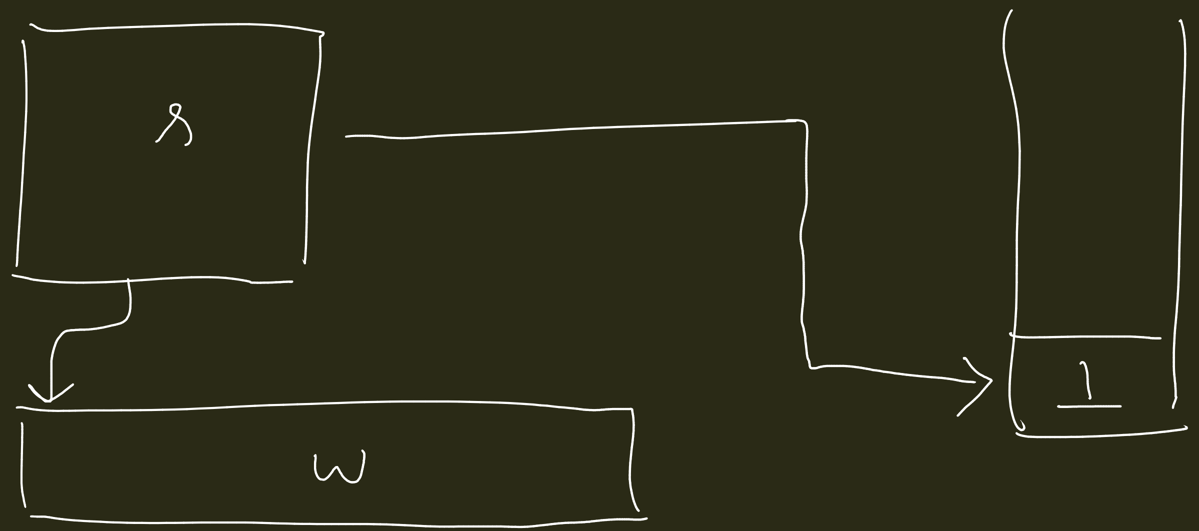
F: a set of final states

$$\delta \subseteq (Q \times \Sigma \cup \{\epsilon\} \times \Gamma) \times (Q \times \Gamma^*)$$

finite

M cannot proceed with the stack empty

Start



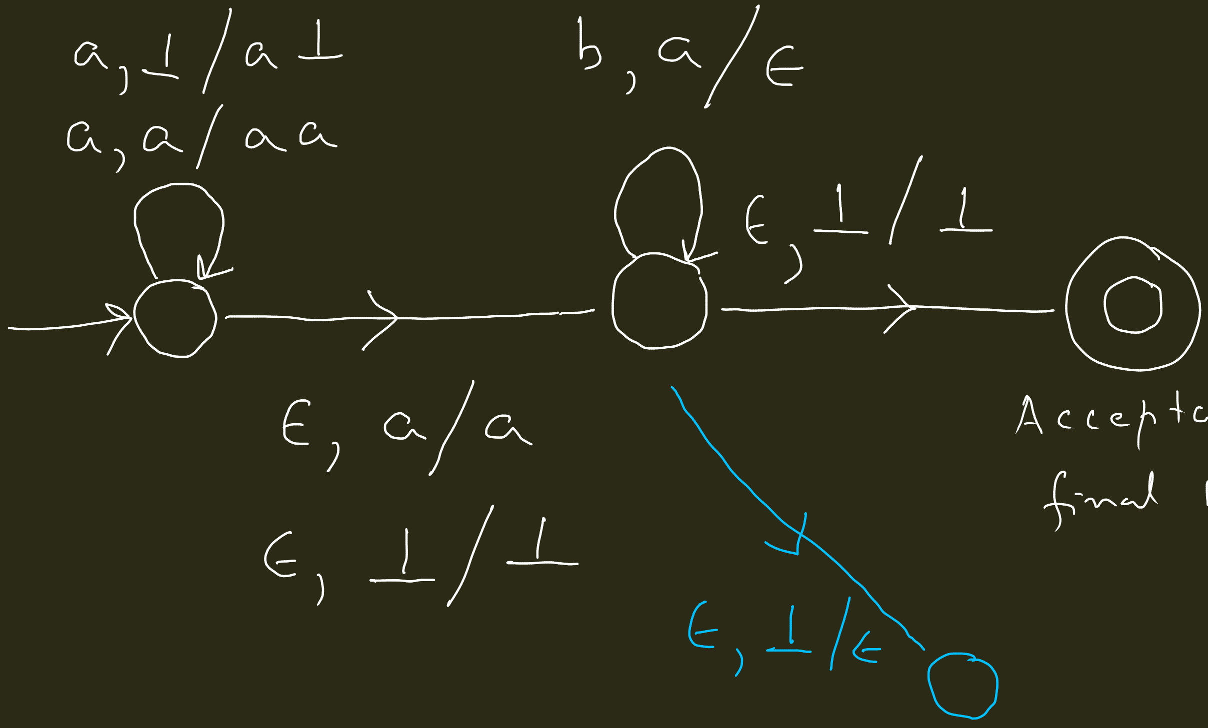
Final

two modes of acceptance / (rejection)

1. by entering a final state
   (stack may contain anything)
2. by emptying the stack ($F = \emptyset$)

Example 1

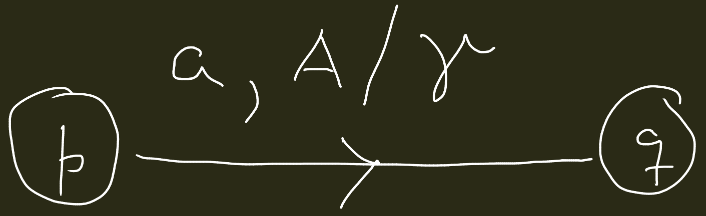
$$\{a^n b^n \mid n \geq 0\}$$



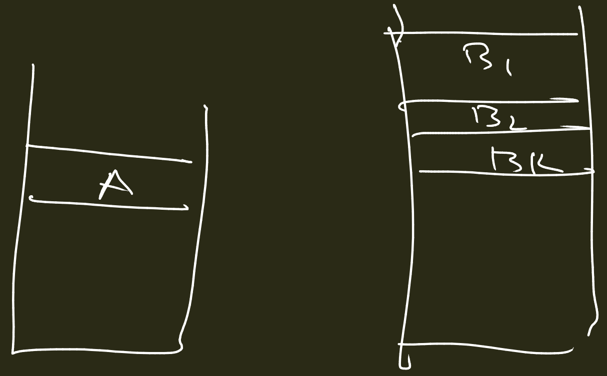
Acceptance by final state

Acceptance by empty stack

$$((p, a, A), (q, \gamma))$$



$$\gamma = B_1 B_2 \cdots B_k$$



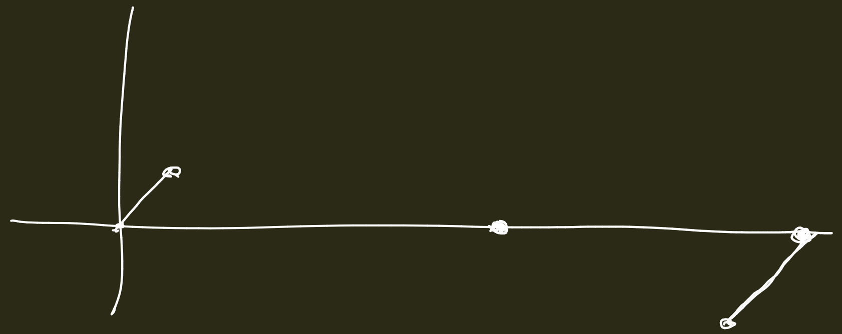
Example 2

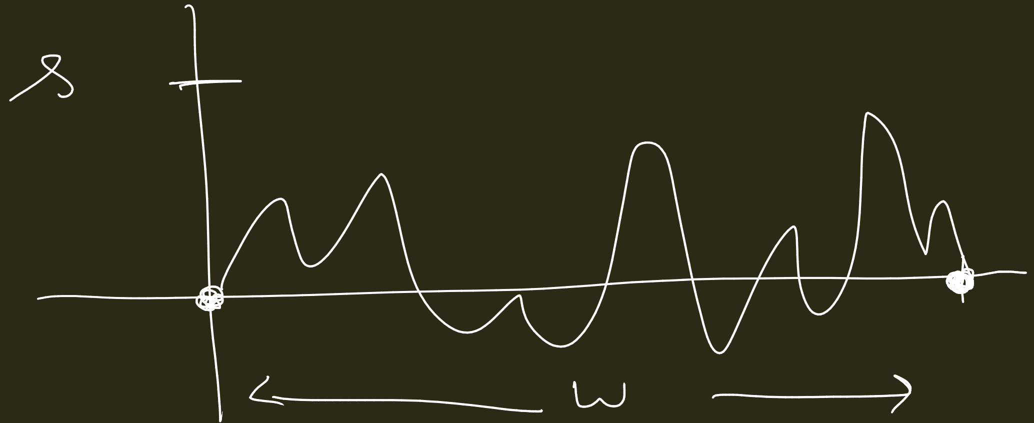
$$\{ w \in \{a, b\}^* \mid \#a(w) = \#b(w) \}$$

CFG:

$$S \to \epsilon \mid aSb \mid bSa \mid SS$$

$$s(x) = \#a(x) - \#b(x)$$





$w = a\,y\,b$ $\qquad$ $w = b\,y\,a$

abba