Given $L \subseteq \Sigma^*$ and $x \in \Sigma^*$, decide whether $x \in L$ or not.

Finite representation of L

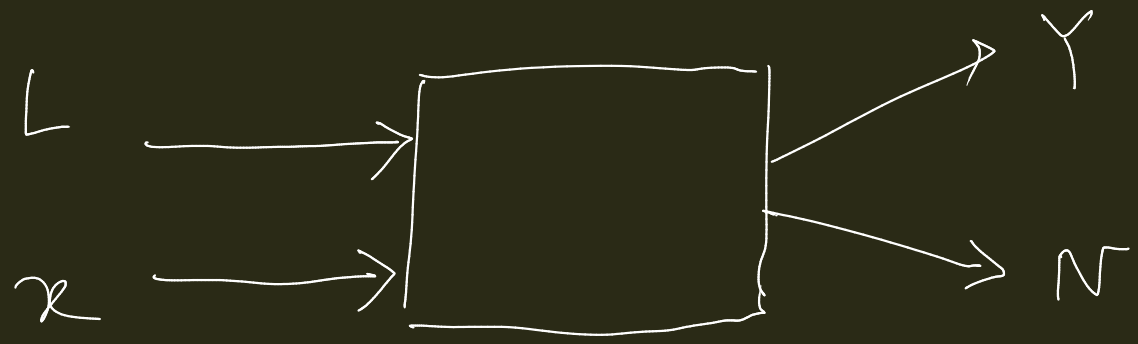
- English language → an alphabet Γ
- Mathematical description
- Recursive description (grammar)
- Machines (automata)

Description is a string over Γ

$\Sigma^* = \bigcup_{n \geq 0} \Sigma^n$ — countable

Γ^* is also countable

How many languages 2^{Σ^*} → uncountable



$\{(L, x)\}$ is countable. — Computers can solve only countably many problems.

having finite rep

— There are uncountably many problems.

"COMPUTERS CANNOT SOLVE ALL PROBLEMS"

There must be unsolvable problems.

There must be unrepresentable numbers.

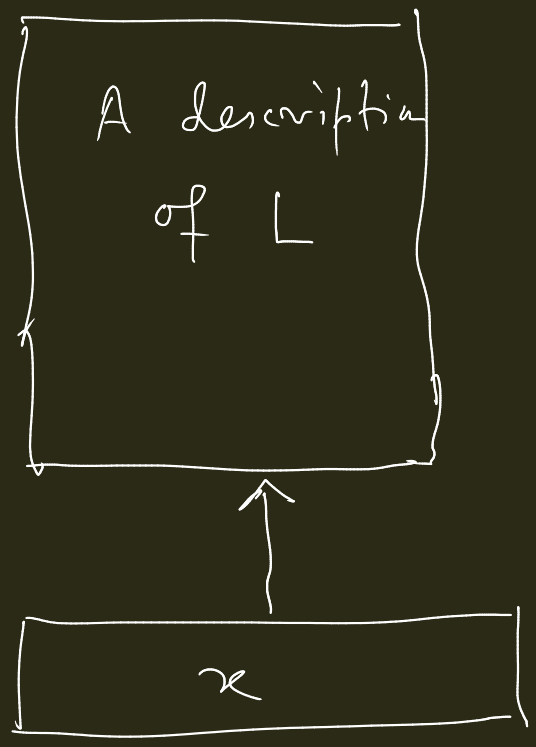
$$\Sigma = \{0, 1, 2, 3, \ldots, 9, \pi, e, \sqrt{}, \sqrt[3]{}, \sqrt[4]{}, \ldots, !, +, - \}$$

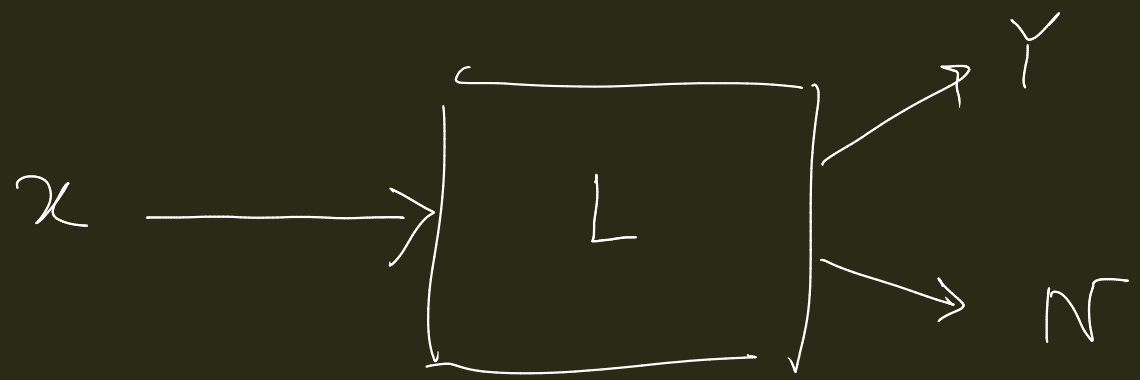
Even if Σ is countably infinite,

$$\Sigma^* = \bigcup_{n \geq 0} \Sigma^n \text{ is countable.}$$

$\pi = 3.14159265358\ldots$

universal Turing machine





can solve only the membership problem for L