Given $L \subseteq U$ and $x \in U$, decide whether $x \in L$ or not.

Representation issues

Elements of U — Each element has a finite representation

$\mathbb{Z}$, $\mathbb{N}$, $\square$, $\$$, $\mathbb{Q}$, $/\!\!A$ — finite

$\pi = 3.14159265358\ldots$

e

$\mathbb{R}$ float double

Every representation can be encoded as a string over some alphabet.

$\mathbb{N} = \{1, 2, 3, \ldots\}$

$\mathbb{N}_0 = \{0, 1, 2, 3, \ldots\}$

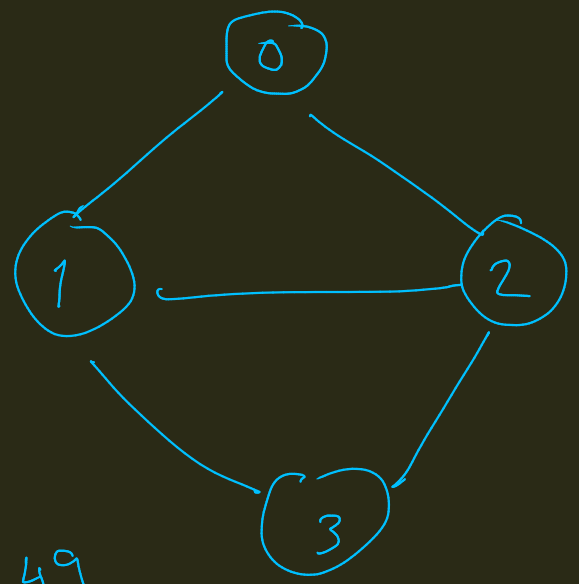
$\mathbb{N} \rightarrow \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$\mathbb{Z} \rightarrow \{+, -\} \cup \{0, \ldots, 9\}$

$\mathbb{Q} \rightarrow \mathbb{Z} \times \mathbb{N}$

$\rightarrow \{+, -, /\} \cup \{0, \ldots, 9\}$

Graphs



4 # 0 1 . . . . . .

row-major rep of the adj matrix

$\{0, 1, \ldots, 9\} \cup \{\#\}$

48 49

4 # 0,1 # 0,2 # 1,2 # 1,3 # 2,3 # #

$\{0, 1, \ldots, 9\} \cup \{\#, ,\}$

Every thing has a representation as a string

We take Σ to be an alphabet

→ a finite set of symbols.

Σ^n = the set of all strings of length n over Σ

$\Sigma^0 = \{\epsilon\}$ → the empty string

$\Sigma^* = \bigcup_{n \geq 0} \Sigma^n$

"Always taking $\Sigma = \{0,1\}$ suffices"

Universal set $U = \Sigma^*$

invalid encoding → some fixed object

Languages are subsets of Σ^*.

Σ To represent $L \subseteq \Sigma^*$

Operation on strings — concatenation

xy — associative

$\times$ commutative

monoid — identity $\in$

$\times$ inverse

Prefix, suffix, powers

suffix

$\hookrightarrow$ x is called a prefix of y

if $y = xz$ for some $z \in \Sigma^*$.

$y = zx$

$x^0 = \epsilon$, $x^n = x\,x^{n-1}$

$(01)^3 = 010101$

$0^3 1^3 = 000111$

Operations on languages

Languages → Sets

String – a finite ordered sequence of symbols

$\cup$, $\cap$, complements, Δ → all set operations are allowed on languages

Concatenation of languages

$$L_1 L_2 = \{ x_1 x_2 \mid x_1 \in L_1 \text{ and } x_2 \in L_2 \}$$

$$L^0 \overset{\text{def}}{=} \{\epsilon\}, \quad L^n = L L^{n-1}$$

$L = \{a, ab\}$

$\Sigma = \{a, b\}$

$L^0 = \{\epsilon\}$, $L^1 = L \cdot L^0 = L = \{a, ab\}$

$L^2 = \{a, ab\}\{a, ab\} = \{aa, aab, aba, abab\}$

$$L^* = \bigcup_{n \geq 0} L^n$$

asterate of L

Kleene-star of L

Given $\underline{\underline{L}}$ and $\underline{\underline{x}}$, ...

How to represent languages?

$L \rightarrow$ finite $\{x_1, x_2, x_3, \ldots, x_n\}$

$L \rightarrow$ infinite what to do?

More machine-worthy representations
- classes of machines
- recursive defn using grammars

Any description of L must be finite

$$\mathbb{P} = \{ n \in \mathbb{N} \mid (n > 1) \wedge \forall u, v \in \mathbb{N} [n = uv \rightarrow (u = 1) \vee (v = 1)] \}$$