CS21004 Formal Languages and Automata Theory, Spring 2019–2020

Practice Exercises

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1. Prove that the following problems are undecidable.

(a) Given a Turing machine M, determine whether M writes the blank symbol on at least one input.
(b) Given a Turing machine M, determine whether M writes the blank symbol on all inputs.
(c) Given a Turing machine M, determine whether M overwrites a non-blank symbol by the blank symbol on at least one input.
(d) Given a Turing machine M, determine whether M overwrites a non-blank symbol by the blank symbol on all inputs.

2. Is it decidable whether a single-tape Turing machine on input ε scans some tape cell three or more times?

3. Recursive / not recursive but r.e / non-r.e.? Supply proofs.

(a) $\{M \mid M \text{ halts on } \varepsilon\}$.
(b) $\{M \mid M \text{ halts on some input}\}$.
(c) $\{M \mid M \text{ halts on all inputs}\}$.
(d) $\{M \mid M \text{ halts on no input}\}$.

4. Recursive or not? Give proofs.

(a) $\{M \# w \mid M \text{ is a one-tape Turing machine that never modifies the input}\}$.
(b) $\{M \mid M \text{ contains a useless state}\}$. A state of M is called *useless* if it is never entered on any input. The accept state and the reject state are never called useless.

5. Recursive / not recursive but r.e / non-r.e.? Supply proofs.

(a) $\{M \mid \mathscr{L}(M) = \mathscr{L}(M)^R\}$ (where L^R is the reverse of L).
(b) $\{M \mid \mathscr{L}(M) = \mathscr{L}(M)\mathscr{L}(M)\}$.
(c) $\{M \mid \mathscr{L}(M) = \mathscr{L}(M)^*\}$.

6. Design nondeterministic Turing machines to accept the following languages.

(a) $\{a^m b^{mn} \mid m,n \geqslant 0\}$.
(b) $\{wvw \mid w \in \{a,b\}^*,\ v \in \{a,b,c\}^*\}$.

7. Design unrestricted grammars for the following languages.

(a) $\{a^n b^{n^2} \mid n \geqslant 0\}$.
(b) $\{a^m b^{mn} \mid m,n \geqslant 0\}$.
(c) $\{w \in \{a,b,c\}^* \mid \#a(w) > \#b(w) > \#c(w)\}$.
(d) $\{wvw \mid w \in \{a,b\}^*,\ v \in \{a,b,c\}^*\}$.

8. Let L be a CFL (specified by a CFG or a PDA), and R a regular language (specified by a DFA or an NFA or a regular expression). Which of the following problems is/are decidable? Supply proofs.

(a) Determine whether $L \subseteq R$.
(b) Determine whether $R \subseteq L$.

9. Prove that given a CFG G, the following problems are undecidable.

(a) Determine whether $\mathscr{L}(G)$ contains a string of the form ww.
(b) Determine whether $\mathscr{L}(G) = \mathscr{L}(G)^R$.

10. Prove that the following problems about DFA D, D_1, D_2 over Σ are decidable.

(a) Whether $\mathscr{L}(D) = \emptyset$.
(b) Whether $\mathscr{L}(D)$ is finite.
(c) Whether $\mathscr{L}(D) = \Sigma^*$.
(d) Whether $\mathscr{L}(D_1) = \mathscr{L}(D_2)$.
(e) Whether $\mathscr{L}(D) = \mathscr{L}(D_1)\mathscr{L}(D_2)$.